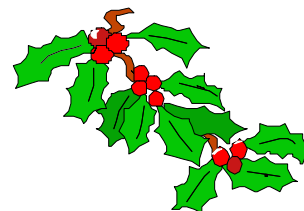




Mathematics Homework

Maths in Action: Higher
Christmas : Units 1 & 2



Show all Working: Use a calculator sparingly!

1. Sketch the parallelogram with vertices A(-4,-2), B(4,0), C(6,4) and D(-2,2).
 - a) Find the equations of the two diagonals AC and BD.
 - b) Find (by algebra) the point of intersection of these two diagonals.

2. A function has the equation $f(x) = 4x^3 - x^4$.
 - a) By setting $y=0$, determine the two roots of this function.
 - b) Find the derivative, $f'(x)$, of this function.
 - c) Solve the equation $f'(x) = 0$, and state the stationary points of this function.
 - d) Draw a fully labelled sketch of this function, clearly indicating all important values.

3. If $p(x) = x^3 + 2x^2 - x - 2$, show that $x+2$ is a factor of $p(x)$. Hence, factorise $p(x)$ FULLY.

4. Use the discriminant to find the value of t , given that the roots of the quadratic $y = 2x^2 + 4x + t$ are equal.

5. Do the following integrations,

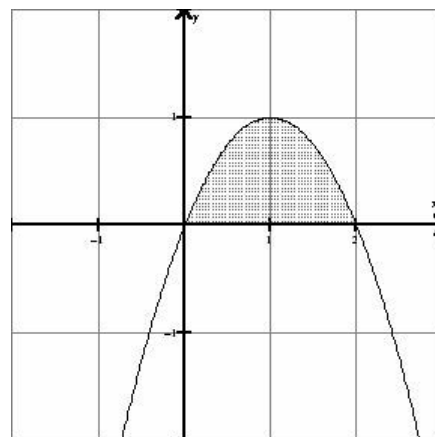
a) $\int (3x^2 + 2x + 1) dx$

b) $\int (1 - 3x)^2 dx$

c) $\int \frac{x^4+1}{x^2} dx$

6. The diagram opposite shows the graph of $y = 2x - x^2$.

Determine the AREA between the curve and the x-axis, as shaded on the diagram.

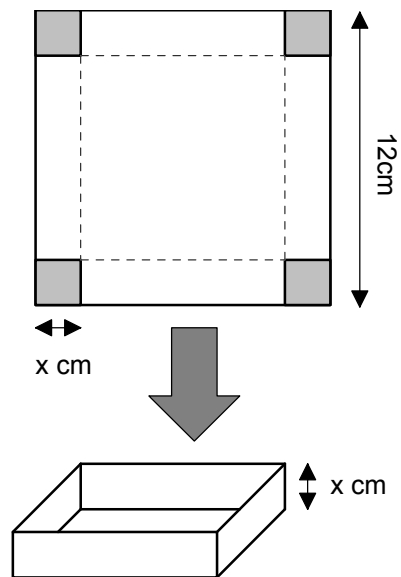


7. In a salmon farm, 25% of the fish are harvested each season. The stock is then immediately topped up by 600 new fish.
 - a) If there are U_n fish at the start of a particular season, write down a recurrence relation for U_{n+1} , the number of fish in the stock at the start of the next season.
 - b) If there were 1500 fish in the farm at the start of the first season, how many fish will be in the farm at the start of the second season?
 - c) The farm has the capacity to hold 2500 fish. Will this always be sufficient to hold all the fish? Explain your answer by calculating a limit to the recurrence relation.

8. a) Make a sketch of the quadratic $y = 2x^2 + 7x - 4$, clearly indicating the roots.
 b) Hence solve the inequation, $2x^2 + 7x - 4 \leq 0$.

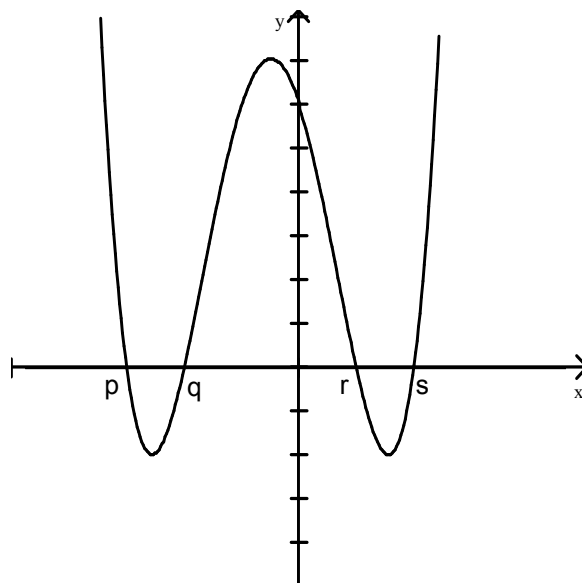
9. Squares of side x cm are cut from the corners of a cardboard square of side 12cm, as shown opposite. The flaps are then folded up to form a small tray.

- a) Show that the volume V cm³ of the tray is $V(x) = 144x - 48x^2 + 4x^3$.
 b) Find the value of x which gives the largest volume of tray and calculate this volume.



10. The diagram opposite shows the graph of $f(x) = x^4 + 2x^3 - 7x^2 + ax + b$

- a) Factorise the quadratic $x^2 + 2x - 3$.
 b) Given that $x^2 + 2x - 3$ is a factor of $f(x) = x^4 + 2x^3 - 7x^2 + ax + b$, find a and b .
 c) Hence find the values of p, q, r and s , the roots of the polynomial, $f(x)$.



11. Solve the equation $3 \cos 2x^\circ + \sin x^\circ - 2 = 0$, for $0 \leq x \leq 360^\circ$
 12. Show that the line $y = 2x - 5$ is a tangent to the circle $x^2 + y^2 + 12x - 6y - 35 = 0$ and find the coordinates of the point of contact.

End of questions. Please hand-in homework on Thursday January 6th 2005.

All diagrams of functions were produced using WinPlot, a free utility available on the internet to download.