Credit Mathematics - Reasoning and Enquiry 2

Non-calculator section:

1. $f(x) = 3x^2 - x + 1$ and $g(x) = x + 9$.
   Find x given $f(x) = g(x)$.

2. $f(x) = 5x^2 - 3x$ and $g(x) = 2x + 3$.
   Find x given $f(x) = 4g(x)$.

3. $P = \dfrac{x^2 + 8x}{4}$. Find x when P = 5.

4. In mathematics one series of numbers is called triangular numbers.
   Any number in the series can be found using the formula

$$T = \frac{n(n+1)}{2}$$

   where T is the triangular number and n is its term in the series.

   The triangular number 45 is which term in the series.

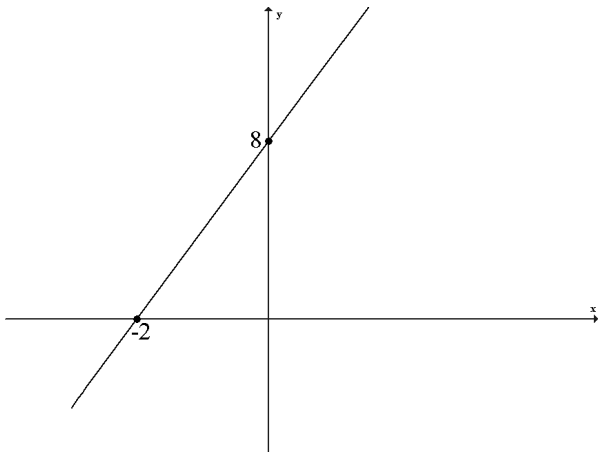
5. John, David and Stephen are brothers. David is 6 years older than John and Stephen is twice as old as John.
   (a) Using x to represent John's age, write down expressions for David and Stephen's ages.
   (b) The sum of the ages of the three brothers is 50. Form an equation in x and use this equation to find John's age.

6. Amanda, Nadia and Laura are sisters.
   Amanda is 4 years younger than Nadia and Laura is 5 years older than Nadia.
   (a) Using x to represent Nadia's age, write down expressions for Amanda's and Laura's ages.
   (b) The sum of their ages is 43. How old is Amanda?

7. A is the point (2,0) and B is (6,8).
   (a) Find the gradient of the line joining A and B.
   (b) If this line is extended it passes through the point (0,-4). Write down the equation of this line.
   (c) Does the point (-5,-12) lie on this line?
   (d) The point (m,3m) lies on this line. Find m.

8. (a) Find the equation of the line opposite..

   (b) The point (n,6n) lies on this line. Find n.



9. A delivery company promises next day delivery on all packages. The cost of delivery is £4.80 for packages weighing up to 10kg plus 40p for each extra kilogram above 10kg.

(a) Find the cost of delivery for a package weighing 16kg.
(b) Write down a formula for the cost, £C, of delivering a package weighing D kilograms, where $D > 10$.

10. Asma hired a mobile phone at a fixed charge of £17 per month. She is also charged for her total call time each month.
The first 25 minutes of her call time is free. The rest is charged at 30p per minute.

(a) Calculate Asma's total cost in a month where she makes 62 minutes of calls.
(b) Write down a formula for the total cost, £C, for Asma's phone in a month where her total call time is t minutes, where $t > 25$.

11. Two different firms charge the following for car hire:

Reliabilty cars: £13.50 per day plus a deposit of £30

Trustmotors : £100 for the first 5 days and then £10 per day after the first five days.

(a) Write down a formula for hiring a car from Reliability cars for **d** days.
(b) Write down a formula for hiring a car from Trustmotors for **d** days where $d > 5$.
(c) Which firm is cheaper?

12. The sequence of odd numbers is 1,3,5,7,……

A pattern for the odd numbers can be written as follows

The **first** odd number: $1 = 1^2 - 0^2$
The **second** odd number: $3 = 2^2 - 1^2$
The **third** odd number: $5 = 3^2 - 2^2$

(a) Express the **fourth** odd number in the same way.
(b) Express 19 in the same way.
(c) Write down a formula for the **n**th odd number. **Simplify this expression.**

13. By adding consecutive terms the following pattern is obtained

$$2 + 8 = 3 \times 2^2 - 2$$
$$2 + 8 + 14 = 3 \times 3^2 - 3$$
$$2 + 8 + 14 + 20 = 3 \times 4^2 - 4$$

(a) Write down the sum of the first 12 terms in the same way.
(b) Write down an expression for the sum of the first n terms.

14. $f(x) = 2^x$. Given $f(x) = \sqrt{32}$ find x.

15. Given $16^{\frac{3}{4}} = 2^x$, find x.

16. Two classes, A and B, sat the same test. Their marks were

Class A:	32	24	45	46	44	31	26	18	19	24	38	40
Class B:	21	25	13	17	40	32	12	24	36	39	22	19

Draw an appropriate diagram to compare these 2 sets of data

Calculator section:

17. Jess has 12 pets, some cats and some dogs.
She insures her pets against illness.
The cost of insurance is £5 for each cat and £7 for each dog.
The total insurance bill is £68.
Let x represent the number of cats Jess has and y the number of dogs.
Form two equations involving x and y and use your equations to find how many dogs Jess has.

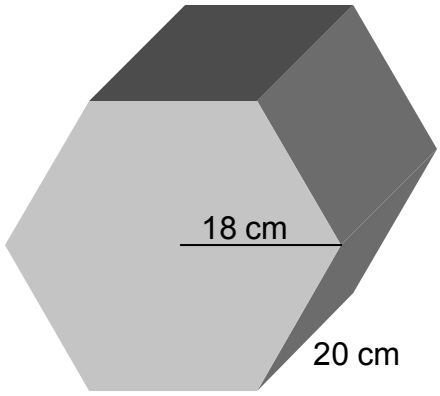
18. A school party is going on an overnight theatre trip to London.
They go to a hotel and book a number of single rooms and a number of twin rooms. In total they book 17 rooms.

Let x represent the number of single rooms booked and y the number of twin rooms booked.
(a) Write down an equation involving x and y.

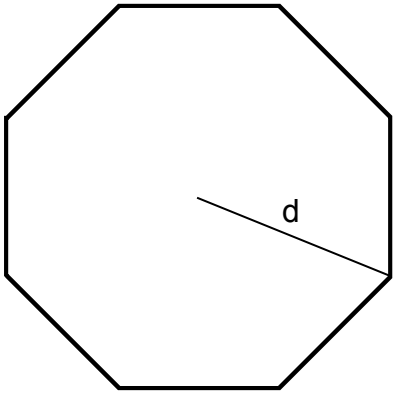
Single rooms cost £35 per night and twin rooms cost £50 per night. The total cost of booking the rooms is £775.
(b)Write down another equation involving x and y.
(c) How many single rooms and how many twin rooms were booked?

19. The diagram opposite shows a prism with a cross-section in the shape of a regular hexagon.

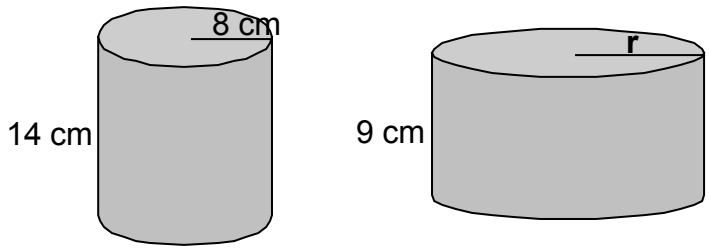
Calculate the volume of this prism.



20. A regular octagon has area 407.29 cm^2 .
Calculate d.

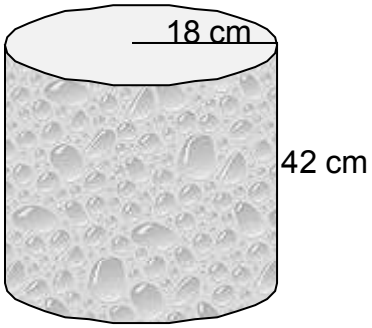


21. The cylinders opposite have the same volume.
Calculate the radius of the second cylinder.



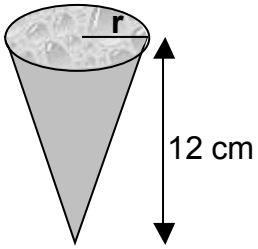
22. (a) A cylindrical water container has radius 18 cm and height 42 cm.

Calculate the volume of this container.



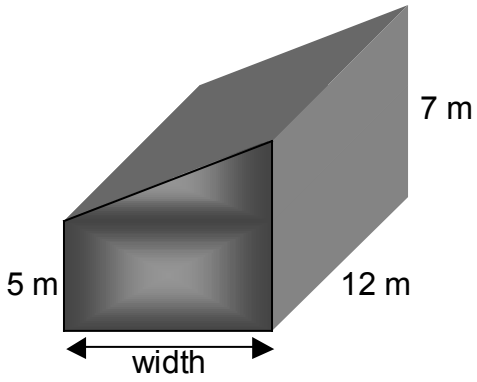
(b) When full, this container can be used to fill 200 cone shaped cups like the one shown.

Calculate the radius of this cup.
Volume of cone = $\frac{1}{3}\pi r^2 h$



23. The prism opposite has volume 612 m^3.

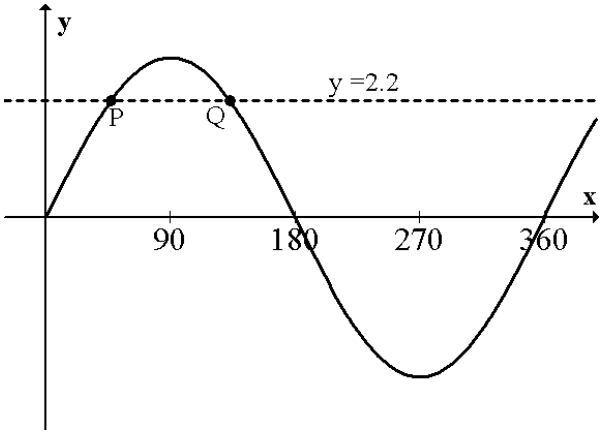
Calculate the width of this prism.



24. A triangle ABC has area 50 cm^2. AB = 12 cm and AC = 16 cm.
Find two possible sizes for angle BAC.

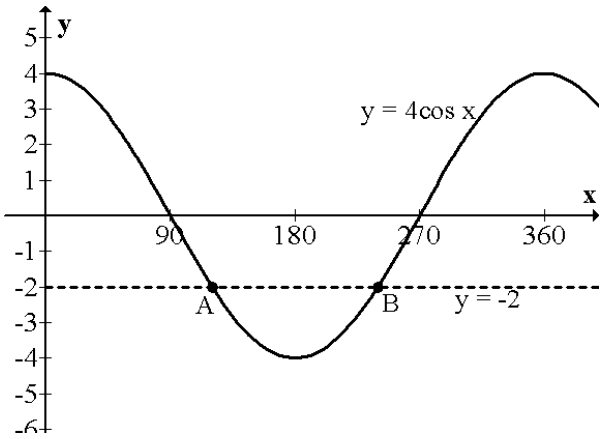
25. A triangle PQR has area 110 cm^2. PQ = 25 cm and QR = 18 cm.
Find the size of angle PQR given it is an obtuse angle.

26. The diagram below shows the graph of y = 3sin x.



The line y = 2.2 has also been drawn on the graph.
Find the coordinates of P and Q.

27. Find the coordinates of A and B in the diagram below.



28. The height of a fairground ride, in metres, is given by the formula

$$H = 6.8 + 3.2\sin(20t)^\circ$$

where t is the time in seconds after the ride starts.

(a) What is the maximum height of the ride?
(b) What is the height of the ride before it starts?
(c) Find the height of the ride after 30 seconds.
(d) After how many seconds does the ride first reach a height of 5.2 metres?

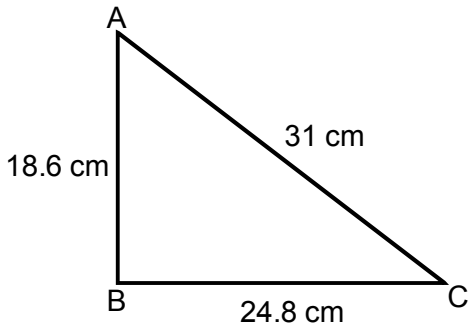
29. The depth of water, D metres, in a harbour is given by the formula

$$D = 14 + 7\sin(15t)^\circ.$$

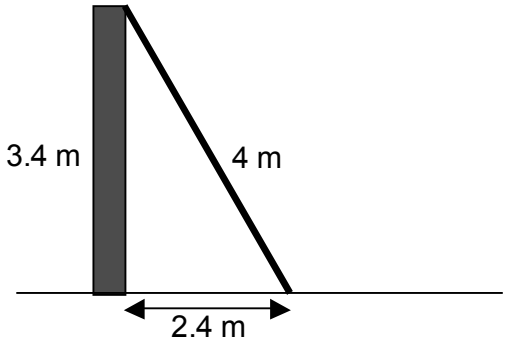
where t is the number of hours after midnight.

(a) What is the maximum depth of water in the harbour?
(b) Calculate the depth of water in the harbour at 2.30pm.
(c) At what **two** times is the depth 10.5m?

30. Show that the triangle opposite is right angled at B.

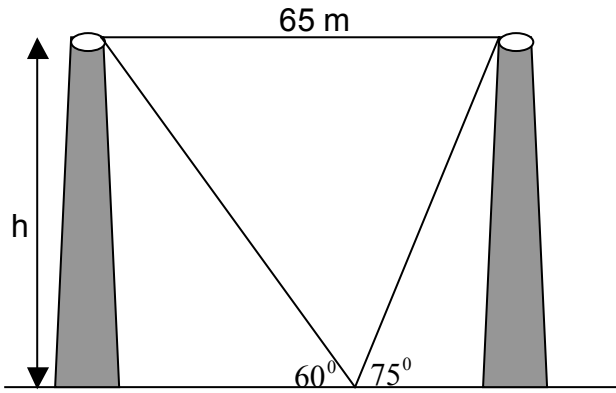


31. A ladder 4 metres long is placed against a wall with the foot of the ladder 2.4 metres from the base of the wall. The ladder reaches the top of the wall which is 3.4 metres high.
Is the wall perpendicular to the ground?



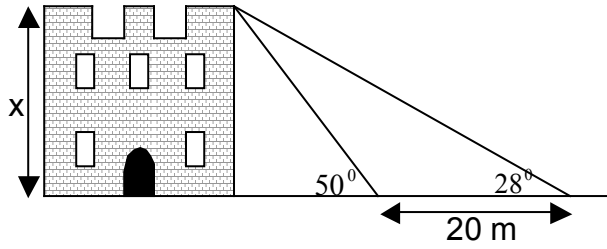
32. Two identical industrial chimneys are 65 metres apart. From a point between the chimneys the angles of elevation to the top of the chimneys are 60^0 and 75^0.

Calculate h, the height of the chimneys.

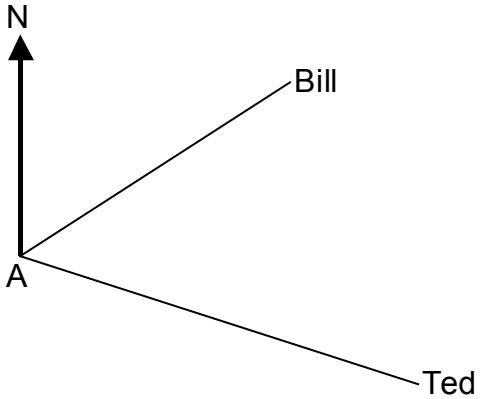


33. From two points 20 metres apart the angles of elevation to the top of a tower are 28^0 and 50^0.

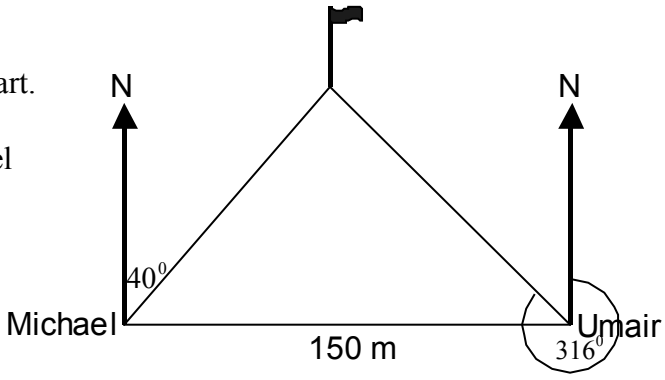
Calculate x, the height of the tower.



34. Bill and Ted leave checkpoint A on a cross-country trek.
Bill walks on a bearing of 055º at a speed of 4kmph.
Ted walks on a bearing of 115^0 at a speed of 3.5kmph.
After 3 hours Bill stops walking but Ted walks for 4 hours before stopping.
How far apart are Bill and Ted after 4 hours?



35. Michael and Umair are standing 150 metres apart. Umair is due East of Michael.
A flagpole is on a bearing of 040° from Michael and on a bearing of 316° from Umair.
How far is Michael from the flagpole?



36. A painting valued at £350000 in the year 2006 rose in value to £367500 in the year 2007.

(a) Find the percentage rise in the value of the painting between 2006 and 2007.

(b) If the painting continues to rise in value at the same percentage rate, find its expected value in the year 2010.

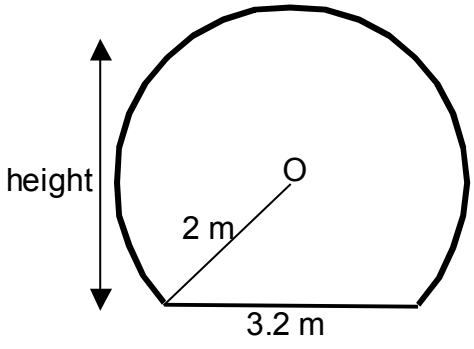
37. A herd of buffalo contained 15000 individuals in 1985. In 1986 the number of buffalo in the herd dropped to 13200.
If the number of buffalo in the herd continues to fall at the same percentage rate, How many will be in the herd after 5 more years?

38. A special offer bottle of shampoo contains 35% extra compared to a normal bottle. If the special offer bottle contains 405 ml, how much does a normal bottle contain?

39. David bought a car in October 2006. By October 2007 the value of the car had fallen by 20%. If the car was worth £6800 in October 2007, what was its value in October 2006?

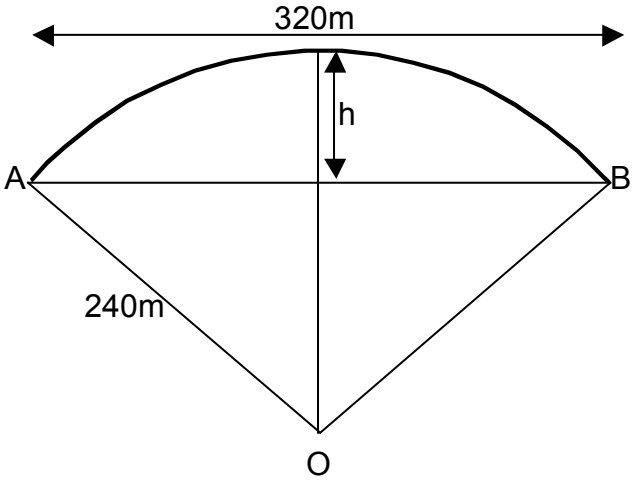
40. The diagram opposite shows an elaborate entrance door to a stately home. The door is in the shape of a circle, centre O, with a straight edge.

Given the information in the diagram, calculate the height of the door.



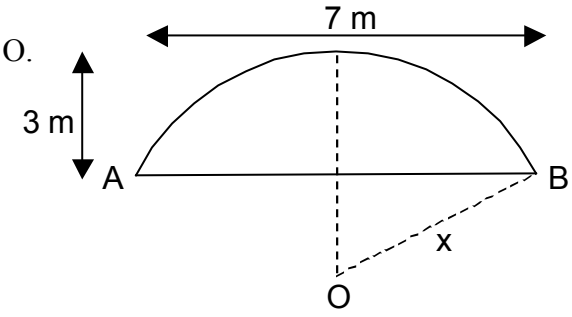
41. The diagram shows the side view of a bridge in the shape of an arc of a circle.
O is the centre of the circle which has radius 240 metres.
AB is 320 metres.

Calculate the distance h.

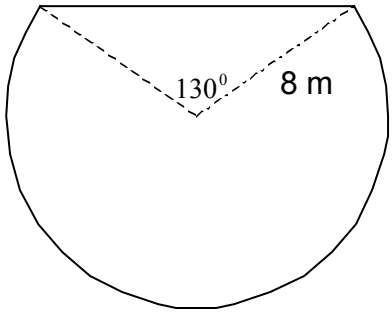


42. The diagram shows a segment of a circle with centre O.
OB is a radius of the circle.

Calculate the length of OB.

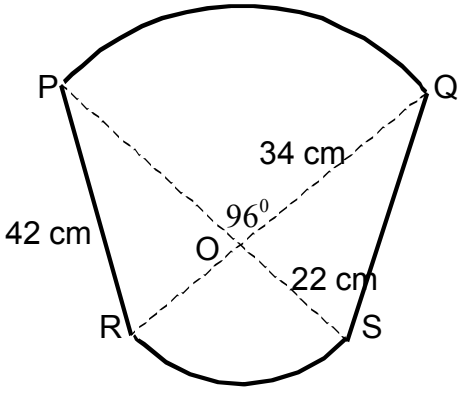


43. The diagram opposite shows a rose garden.
The garden is in the shape of a circle centre O, with a straight edge.
A wall is to be built round the outside of the garden.
How long will the wall be?



44. In the diagram PQ and RS are arcs of circles with centre O.
The radius, OQ, is 34 centimetres long and the radius, OS, is 22 centimetres long.

Calculate the perimeter of the shape.

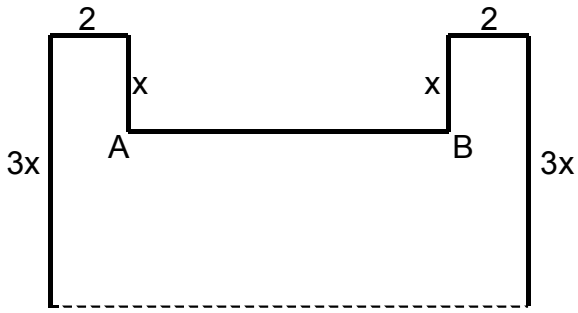


45. A = mn and $8m + 4n - 160 = 0$

(a) Show that $A = 40m - 2m^2$

(b) Find the maximum value of A and the corresponding value of m.

46. The perimeter of the shape opposite is 38 metres.

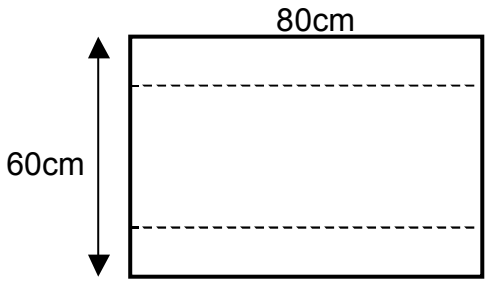


(a) Show that $AB = 34 - 8x$

(b) Hence show that the area of the shape is $A = 80x - 16x^2$.

(c) Find the value of x for which the area of the shape is maximised and find this maximum area.

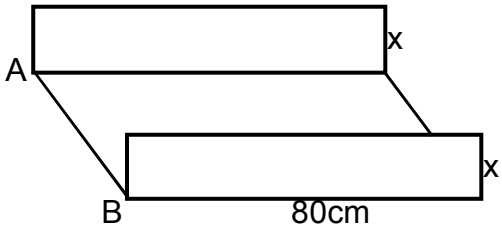
47. A rectangular sheet of metal 40 centimetres by 60 centimetres is folded as shown opposite.



(a) Write down the length of AB

(b) Show that the volume of the shape is given by $V = 4800x - 160x^2$.

(c) Find the value of x for which the volume is maximised and find the volume when x takes this value.



48. A rectangular garden pond is 10 metres in length and 8 metres wide.
The length of the pond is to be increased by 2x metres and the width by x metres.

(a) Write down the new length and the new width of the pond.

(b) If the new pond has area 176 square metres, show that

$$2x^2 + 26x - 96 = 0$$

(c) Find the value of x.