

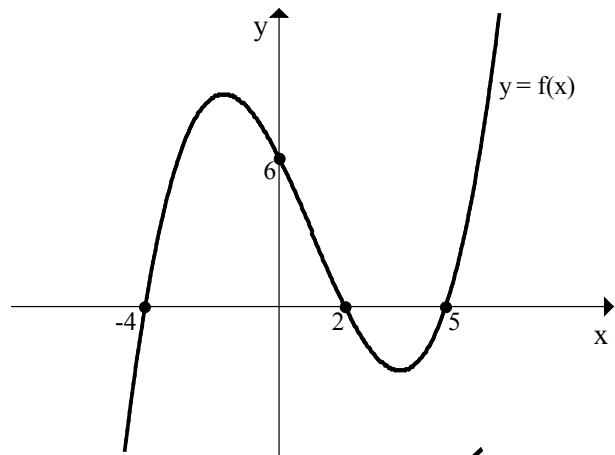
## Higher Maths – Homework 10

### Non-calculator section:

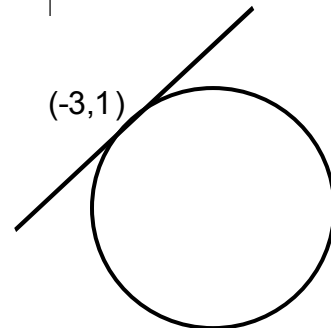
1. (a) Given  $(x + 2)$  is a factor of  $f(x) = x^3 - px - 6$ , find  $p$ .  
(b) Hence factorise  $f(x)$  fully.
2. (a) A is the point  $(3,1,-4)$  and C is  $(15,13,-16)$ . B divides AC in the ratio 2:1, find the coordinates of B.  
(b) D is the point  $(12,11,-10)$  and E is  $(8,3,-18)$ . Show that D, B and E are collinear.
3. Given  $\tan x = \frac{1}{2}$ , show that (i)  $\sin 2x = \frac{4}{5}$  (ii)  $\cos 2x = \frac{3}{5}$ .

4. The diagram shows the graph of  $y = f(x)$ .

Sketch the graph of  $y = -f(x - 2)$ .



5. A circle has equation  $x^2 + y^2 - 6x + 8y - 36 = 0$ . Find the equation of the tangent to this circle at the point  $(-3,1)$ .



6. A curve has equation  $y = x^2 - 12\sqrt{x}$ . Find the equation of the tangent to this curve at the point where  $x = 4$ .
7. PQR is a triangle with  $P(2,-1)$ ,  $Q(2,9)$  and  $R(5,0)$ .
  - (a) Find the equation of the perpendicular bisector of PQ.
  - (b) Find the equation of the altitude from P to QR.
  - (c) Find the point of intersection of these two lines.
8. Solve the equation  $\log_3(x + 2) + \log_3 x = 1$ ,  $x > 0$ .

9. Given  $y = 2\sin^2 x + \frac{2}{(x+1)}$  find  $\frac{dy}{dx}$ .

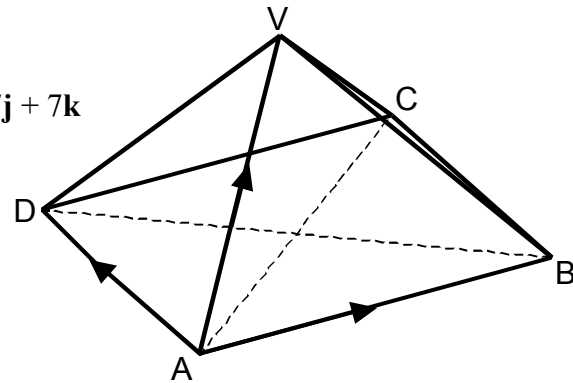
**Calculator section:**

10. VABCD is a pyramid with rectangular base ABCD.

The vectors  $\overrightarrow{AB}$ ,  $\overrightarrow{AD}$  and  $\overrightarrow{AV}$  are given by

$$\overrightarrow{AB} = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} \quad \overrightarrow{AD} = -2\mathbf{i} + 10\mathbf{j} - 2\mathbf{k} \quad \overrightarrow{AV} = \mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$$

- (a) Express  $\overrightarrow{VC}$  in component form.  
 (b) Calculate the size of angle AVC.



11. The diagram opposite shows a parallelogram.

Given  $|\mathbf{a}| = 5$  and  $|\mathbf{b}| = 4$ , show that  $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 35$ .

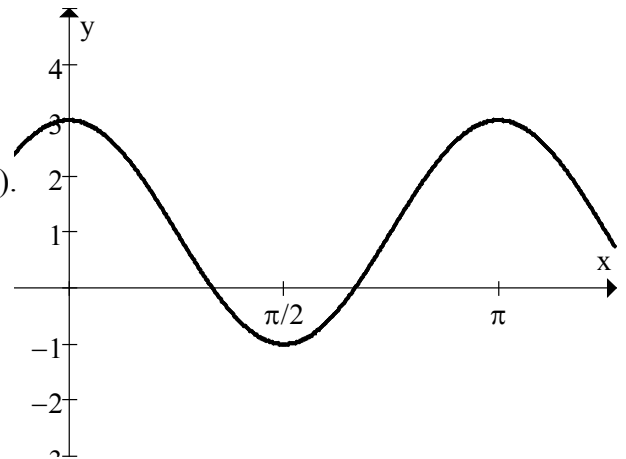
12.  $f'(x) = 6(2x - 1)^2$  and  $f(2) = 17$ .

Find a formula for  $f(x)$ .

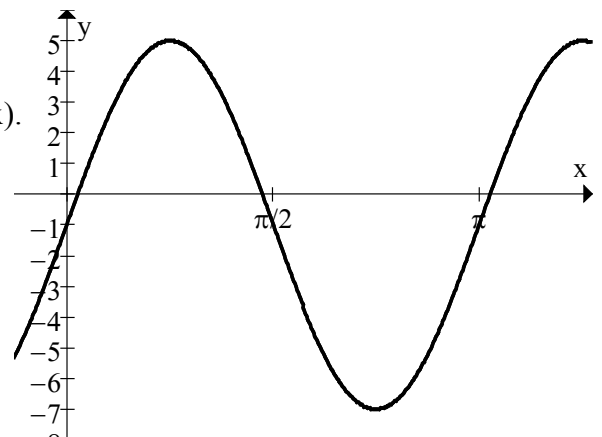
13.  $f(x) = \frac{x+1}{x} - 1$  and  $g(x) = \frac{2}{x}$   $x \neq 0$ .

Show that  $f(g(x)) = \frac{1}{2}x$ .

14. (a) The diagram opposite shows the graph of  $f(x) = a \cos bx + c$ . Write down a formula for  $f(x)$ .



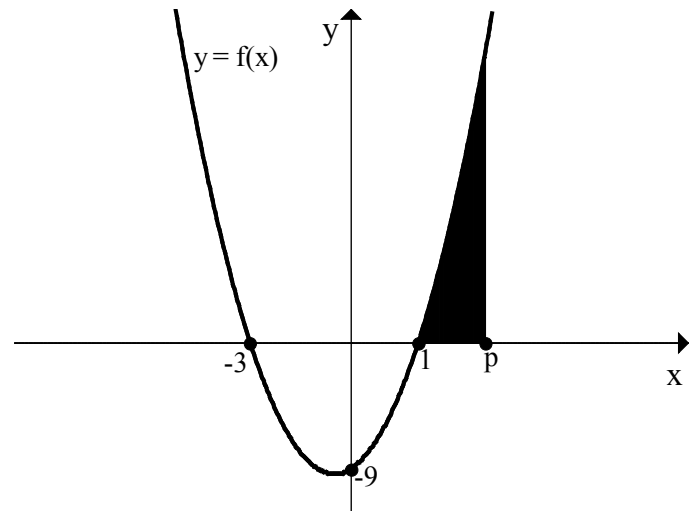
- (b) The diagram opposite shows the graph of  $g(x) = a \sin bx + c$ . Write down a formula for  $g(x)$ .



- (c) Express  $f(x) + g(x)$  in the form  $k \cos(2x - \alpha)$ .

- (d) Hence solve  $f(x) + g(x) = \sqrt{10}$ ,  $0 \leq x \leq 360$

15. (a) The diagram shows the graph of  $y = f(x)$ .  
Find a formula for  $f(x)$ .
- (b) The shaded region has an area of 32 units.  
Find  $p$ .



16. (a) State the condition for a quadratic equation to have equal roots.
- (b) The equation  $(x + k)^2 = k(x - 1) + 1$  has equal roots, find  $k$ .