

Higher Mathematics – Homework G

Non – calculator section:

1. $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$.

- (a) Find the vectors $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$
(b) Show that the vectors $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular.

2. Solve the equation $\log_2 (7x + 4) - \log_2 x = 3$

3. $f(x) = 1 - x$ and $g(x) = 6 - x^2$

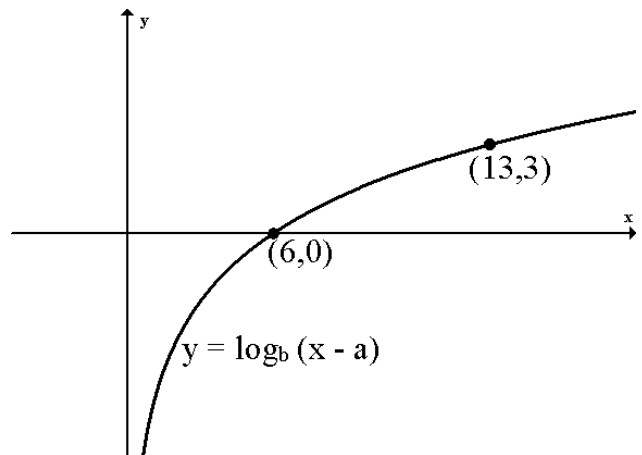
- (a) $h(x) = g(f(x))$. Show that $h(x) = 5 + 2x - x^2$
(b) Express $h(x)$ in the form $a - (x + b)^2$
(c) Hence, or otherwise, write down the turning point of $h(x)$, stating whether it is a maximum or minimum.

4. (a) Express $\cos x + \sqrt{3} \sin x$ in the form $k \cos(x - a)$ where $k > 0$ and $0 \leq a \leq 360$.

(b) Hence solve the equation $\cos x + \sqrt{3} \sin x = \sqrt{3}$ for $0 \leq x < 90$.

5. The diagram shows the graph of $y = \log_b (x - a)$.

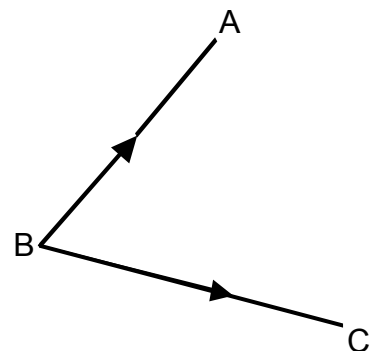
Find a and b .



6. Show that the tangent to the curve $y = 4 \sin(3x - \frac{\pi}{3})$ at the point where $x = \frac{\pi}{6}$ has equation $y - 2 = 6\sqrt{3}(x - \frac{\pi}{6})$

7. A is the point $(-1, 2, 4)$, B is $(0, 4, 2)$ and C is $(-4, 0, 2)$.

Calculate the size of angle ABC.



Calculator section:

8. A recurrence relation is defined as $u_{n+1} = au_n + b$.

(a) Given $u_1 = 32$, $u_2 = 20$ and $u_3 = 17$, find the values of a and b .

(b) The limit of the recurrence relation in part (a) is the same as the limit of $v_{n+1} = pv_n + 10$. Find the value of p .

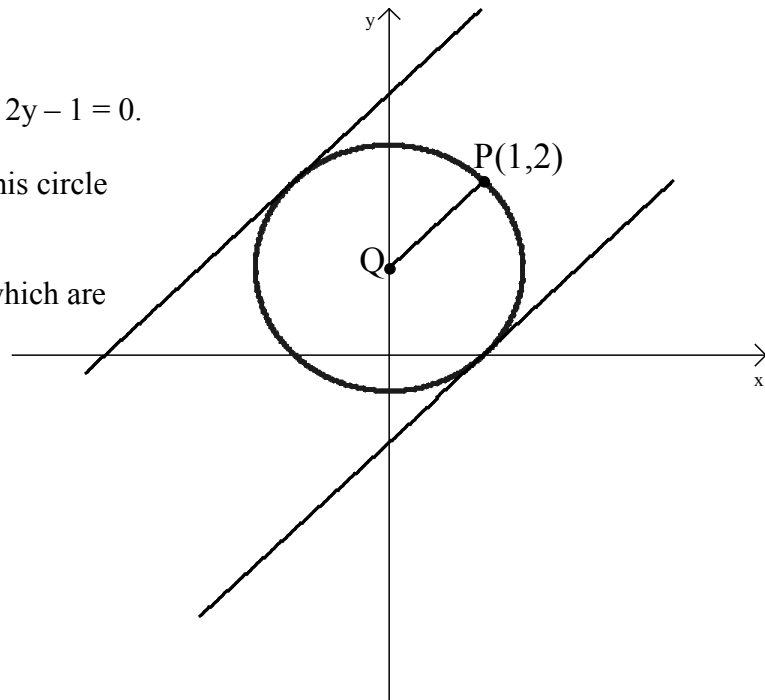
9. Find the values of x for which the function $f(x) = \frac{1}{6}(2x - 3)^3 - x$ is increasing.

10. Solve the equation $2\cos 2x - 1 = \cos x$ $0 \leq x \leq 360$

11. A circle, centre Q , has equation $x^2 + y^2 - 2y - 1 = 0$.

(a) Find the equation of the tangent to this circle at the point $P(1,2)$.

(b) There are two tangents to the circle which are parallel to the radius PQ . Find the equations of these tangents.



12. (a) The parabola opposite cuts the x -axis at -1 and p and the y -axis at $-2p$. Show that the parabola has equation $y = 2x^2 + 2x(1 - p) - 2p$.

(b) The shaded area has a value equal to $\frac{-10p}{3}$.

Calculate the value of p .

