

Higher Mathematics
Homework – Differentiation

1. $f(x) = \frac{2x - x^2}{\sqrt[3]{x}}$. Show that $f'(8) = -6$.

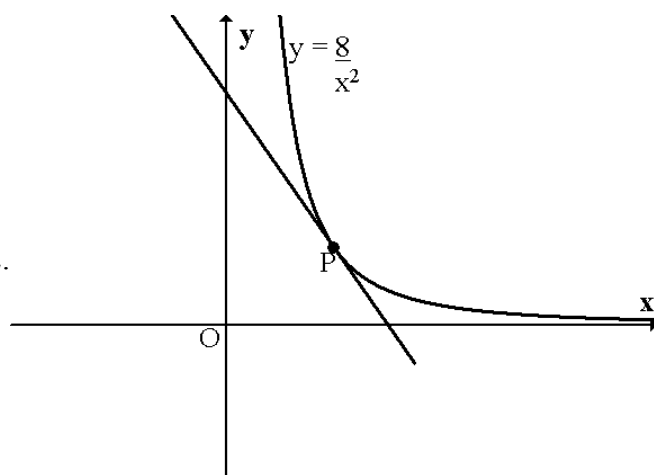
2. The position of an atomic particle, relative to its starting position, is given by $s(t) = 2t^3 + 4t^2 - 3t$, where t is the time in seconds and $s(t)$ is the distance in centimetres.

Calculate

- (a) the position of the particle after 2 seconds.
- (b) the speed of the particle after 5 seconds.

3. The diagram shows part of the graph of $y = \frac{8}{x^2}$.

Find the equation of the tangent at P, where $x = 2$.



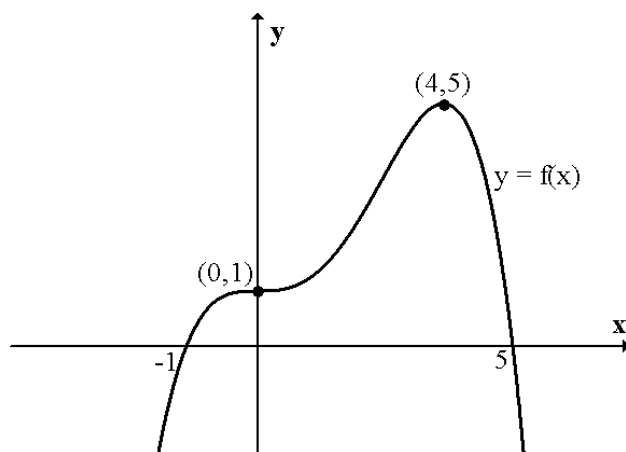
4. Show that the function $f(x) = -x^3 + 6x^2 - 12x + 4$ is never increasing.

5. A function has equation $f(x) = \frac{1}{2}x^4 - 2x^3 + 4$.

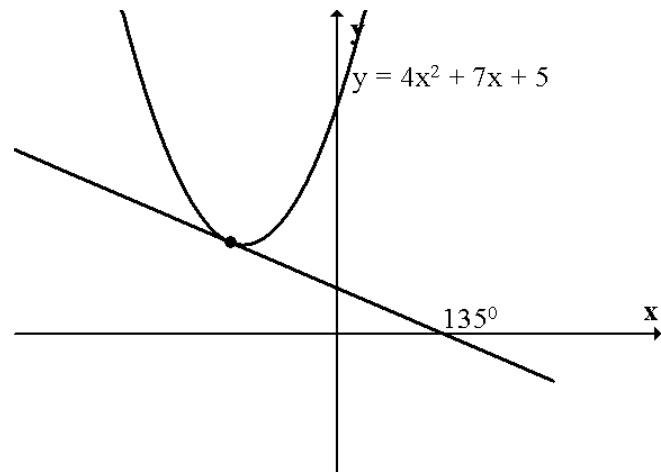
Find the stationary points of $f(x)$ and determine their nature.

6. Find the intervals in which the function $f(x) = x^3 + 3x^2 - 24x + 2$ is increasing.

7. The diagram opposite shows the graph of $y = f(x)$.
Sketch the graph of $y = f'(x)$.



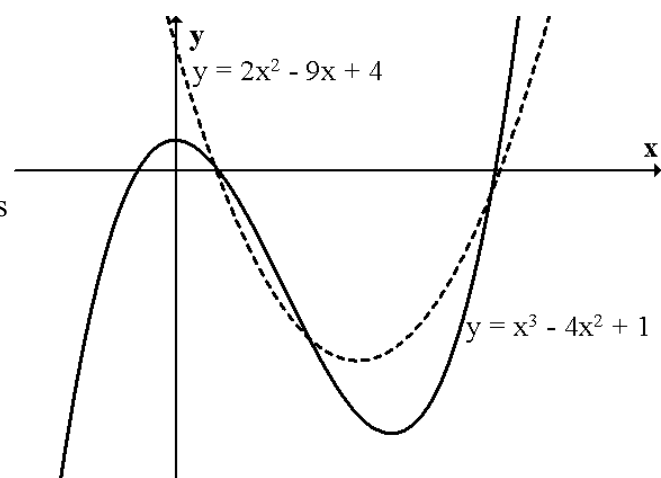
8. A tangent to the curve $y = 4x^2 + 7x + 5$ makes an angle of 135° with the positive direction of the x-axis.
Find the equation of this tangent.



9. Find the x-coordinates of the points on the curves

$$y = x^3 - 4x^2 + 1 \quad \text{and} \quad y = 2x^2 - 9x + 4$$

where the gradients are equal.



10. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$.
Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.