

D321 12, D322 12 (units 1 and 2)

Preparing for exams by

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Qualification

Authority

Mathematics
Higher Grade
Prelim Exam
2008

Paper 1

Non-calculator paper

1 hour 30 minutes

Read Carefully

1. Calculators may not be used.
2. Section A - Questions 1-20 (40 marks)
Instructions for the completion of section A are given on the next page.
For this section of the examination you should use an HB pencil.
3. Section B - Written response questions (30 marks)
Full credit will be given only where the solution contains appropriate working.
Answers obtained by reading from scale drawing will not receive any credit.



KNIGHTSWOOD SECONDARY
MATHEMATICS DEPARTMENT
GLASGOW



Read Carefully

1. Check that the answer sheet provided is for **Mathematics Higher Prelim 2008**.
2. For this section of the exam you must use an **HB pencil** and, where necessary, an eraser.
3. Make sure you write your **name, class** and **teacher** on the answer sheet provided.
4. The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5. There is only **one correct answer** to each question.
6. Rough working should **not** be done on your answer sheet.
7. Make sure at the end of the exam that you hand in your answer sheet for section A with the rest of your written answers.

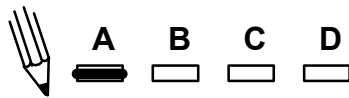
Sample question

A line has the equation $y = 4x - 1$.

If the point $(k, 7)$ lies on the line, the value of k is

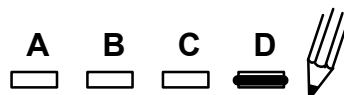
- A** 2
- B** 27
- C** 1.5
- D** -2

The correct answer is A \rightarrow 2. The answer A should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to D.



Formulae List

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

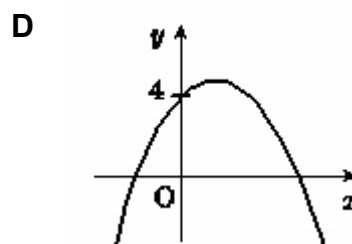
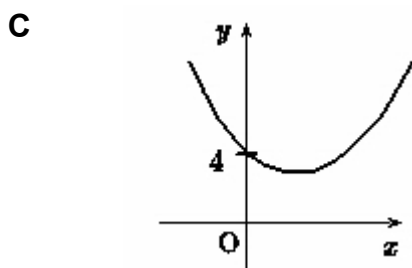
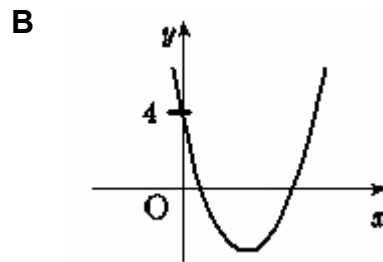
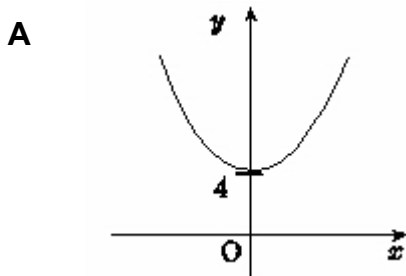
The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

Section A
All questions should be attempted

1. P and Q are the points (2,3) and (-1,4).
What is the gradient of the line perpendicular to PQ?
- A** $-\frac{8}{7}$
B 3
C 5
D 7
2. $f(x) = 2x - 1$ and $g(x) = 2x + 1$ are functions defined on the set of real numbers.
Find an expression for $f(g(x))$.
- A** $f(g(x)) = 4x^2 - 1$
B $f(g(x)) = 4x^2$
C $f(g(x)) = 4x$
D $f(g(x)) = 4x + 1$
3. Which of the sketches shown below is most likely to represent the graph of $y = 3x^2 - 7x + 4$?



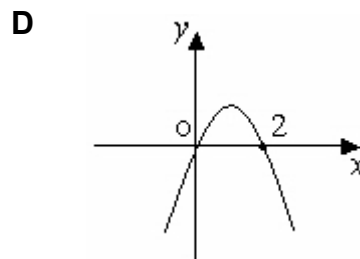
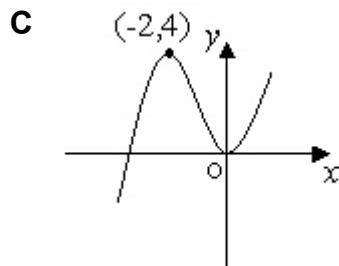
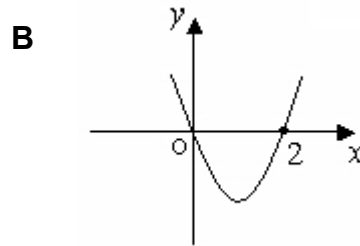
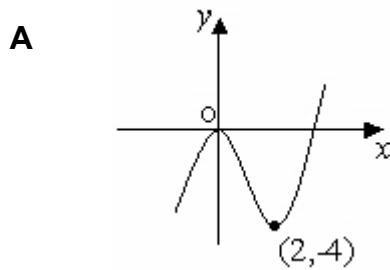
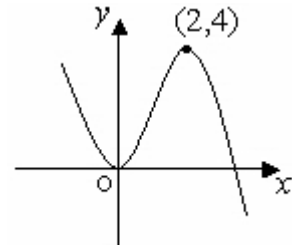
4. If $f(x) = 6x^3 - 2x^{-\frac{1}{2}}$ then the derivative, $f'(x)$ is
- A** $18x^2 + x^{-\frac{3}{2}}$
B $2x^2 + 4x^{\frac{1}{2}}$
C $6x^2 - x^{-\frac{3}{2}}$
D $18x^2 + x^{\frac{1}{2}}$

5. PQ is the diameter of a circle.
P and Q have the coordinates (3,2) and (7,2) respectively.
What is the equation of the circle?

- A** $(x-3)^2 + (y-2)^2 = 16$
B $(x-4)^2 + y^2 = 2$
C $(x+5)^2 + (y+2)^2 = 2$
D $(x-5)^2 + (y-2)^2 = 4$

6. Part of the graph of $y = f(x)$ is shown opposite.

The graph of the derivative, $y = f'(x)$ could be represented by,



7. What is the remainder on dividing the polynomial $5x^3 - 4x + 8$ by $x - 2$?

- A** -24
B 0
C 8
D 40

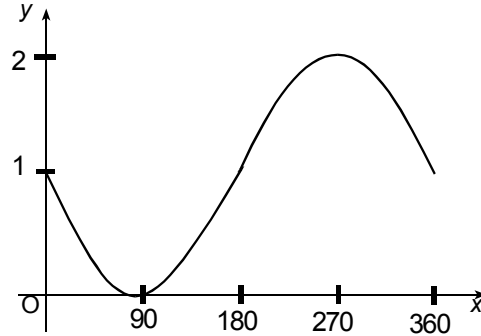
8. Find $\int_{-1}^1 x^4 dx$

- A** 0
B $\frac{1}{4}$
C $\frac{2}{5}$
D 8

9. The quadratic equation $4kx^2 - 8x + k = 0$ has equal roots.
The value of k , where $k > 0$ is,

- A 4
- B 2
- C 0
- D -2

10. The diagram below shows the graph of a trigonometric function.



Which of the following could be the equation of the graph?

- A $y = 1 + \sin x^\circ$
- B $y = 1 - \sin x^\circ$
- C $y = 2 - \cos x^\circ$
- D $y = 2 \cos x^\circ - 1$

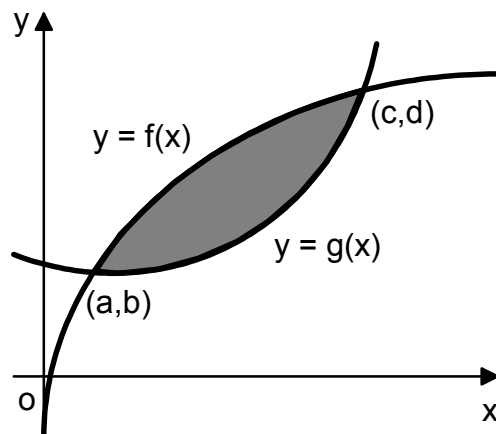
11. A recurrence relation is defined by $U_{n-1} = 0.4U_n - 24$.
The limit of this sequence is,

- A -40
- B -24
- C 0.03
- D 50

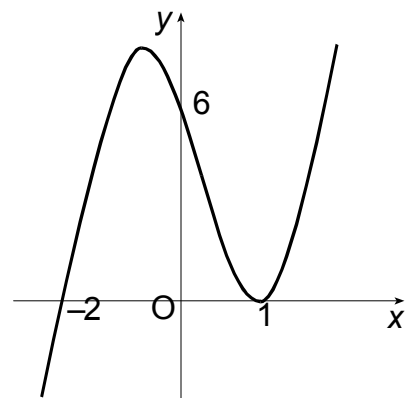
12. The graphs $y = f(x)$ and $y = g(x)$ intersect at points (a,b) and (c,d) as shown opposite.

The shaded area is given by,

- A $\int_b^d (f(x) - g(x)) dx$
- B $\int_a^c (f(x) + g(x)) dx$
- C $\int_a^c (f(x) - g(x)) dx$
- D $\int_a^d (f(x) - g(x)) dx$



13. Given that $\cos x^\circ = \frac{1}{\sqrt{3}}$ and $0 < x^\circ < 90$, then the exact value of $\cos 2x$ will be,
- A** $\frac{\sqrt{3}}{2}$
- B** $-\frac{1}{3}$
- C** $\frac{1}{3}$
- D** $\frac{1}{2\sqrt{3}}$
14. The tangent to the curve with equation $y = 2x^3 - 1$ at the point where $x = 1$ has a gradient of,
- A** 1
- B** 6
- C** 5
- D** $-\frac{1}{2}$
15. A circle has the equation $x^2 + y^2 + 4x - 2y - 4 = 0$. Which of the following correctly states the coordinates of its centre and the value of its radius?
- A** $(-2, 1)$, $r = 1$
- B** $(2, -1)$, $r = 3$
- C** $(-2, 1)$, $r = 3$
- D** $(2, -1)$, $r = 1$
16. When $x^2 + 8x + 5$ is expressed in the form $(x + a)^2 + b$, what is the value of b ?
- A** -59
- B** -11
- C** 0
- D** 5
17. The diagram opposite shows part of a cubic function.
What is the equation of this graph?
- A** $y = 6(x + 2)(x - 1)^2$
- B** $y = 3(x - 2)(x + 1)^2$
- C** $y = 3(x + 2)(x - 1)^2$
- D** $y = 6(x - 2)(x + 1)^2$



18. A line **L**, is parallel to the line with equation $y = -2x + 3$ and passes through the point $(-3, 1)$. What is the equation of **L**?

- A** $y - 1 = -2(x - 3)$
- B** $y - 1 = 4(x - 3)$
- C** $y - 1 = -2(x + 3)$
- D** $y + 3 = -2(x - 1)$

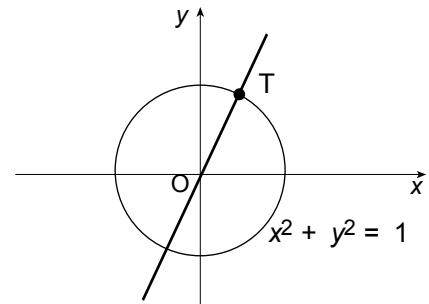
19. Find $\int \frac{1}{5\sqrt{x}} dx$.

- A** $\frac{2}{5}x^{\frac{1}{2}} + c$
- B** $\frac{5}{2}x^{\frac{1}{2}} + c$
- C** $-\frac{1}{10}x^{-\frac{3}{2}} + c$
- D** $\frac{1}{10}x^{-\frac{3}{2}} + c$

20. The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 1$ at the point **T**.

What is the x-coordinate of **T**?

- A** $\frac{1}{3}$
- B** $\frac{1}{\sqrt{6}}$
- C** $\frac{1}{\sqrt{5}}$
- D** $\frac{1}{2}$



Section B
All questions should be attempted

21. A function is given by $f(x) = x^3 + 3x^2 + 2x + 6$.
- a) Show that $(x+3)$ is a factor of $f(x)$. (2)
- b) Hence solve the equation $x^3 + 3x^2 + 2x + 6 = 0$ and clearly state why there is only one real root. (3)

22. Find the equation of the **tangent** at the point $(-2, -1)$ on the circle with equation $x^2 + y^2 - 6x - 4y - 21 = 0$. (5)

23. a) Find the derivative of the function

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + 11$$
 (2)
- b) Construct a nature table and hence determine the values of x for which the function is **decreasing**. (3)

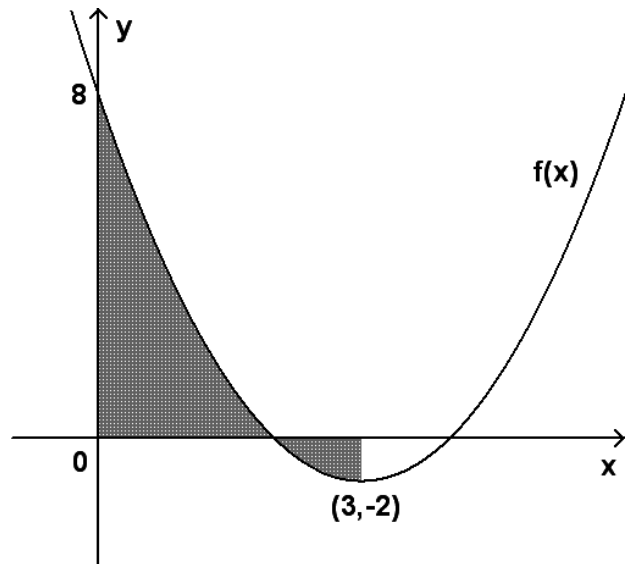
24. Solve algebraically the equation

$$\cos 2x^\circ + 5 \cos x^\circ - 2 = 0 \quad \text{where } 0 \leq x \leq 360^\circ$$
 (5)

25. This is the graph of $y = x^2 - 6x + 8$.

Calculate the area of the shaded region.

(5)



26. If $\sin x = \frac{5}{13}$ and $\tan y = \frac{2}{3}$
- a) Determine $\cos x$, $\cos y$ and $\sin y$. (2)
- b) Hence show that $\cos(x+y) = \frac{2}{\sqrt{13}}$ (3)

End of question paper