

D321 12, D322 12 (units 1 and 2)

Preparing for exams by

Scottish

Qualification

Authority

Mathematics
Higher Grade
Prelim Exam
2008

Paper 2

Calculator paper
1 hour 10 minutes

Read Carefully

- 1 Full credit will be given only where the solution contains appropriate working.
- 2 Calculators should be used and all questions should be attempted.
- 3 Answers obtained by reading from scale drawing will not receive any credit.

Formulae List

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

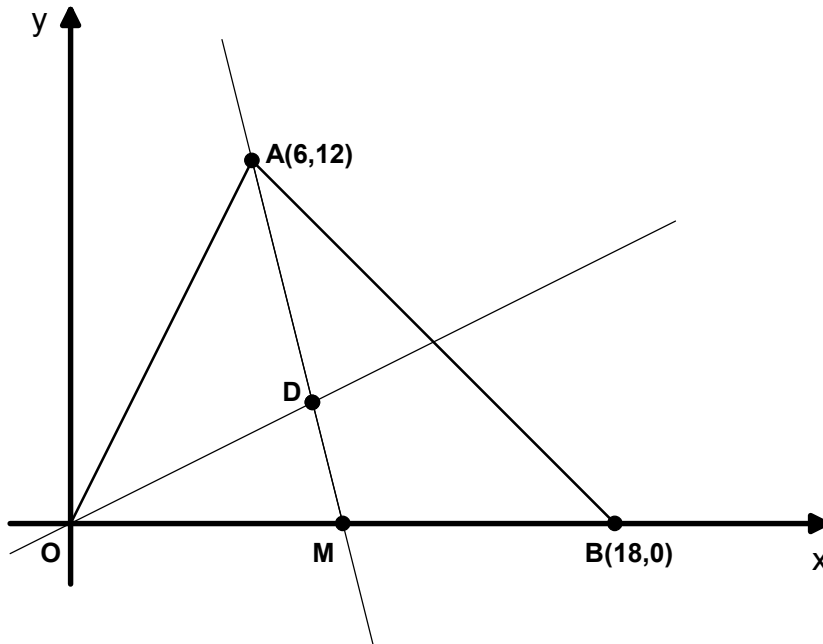
Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$


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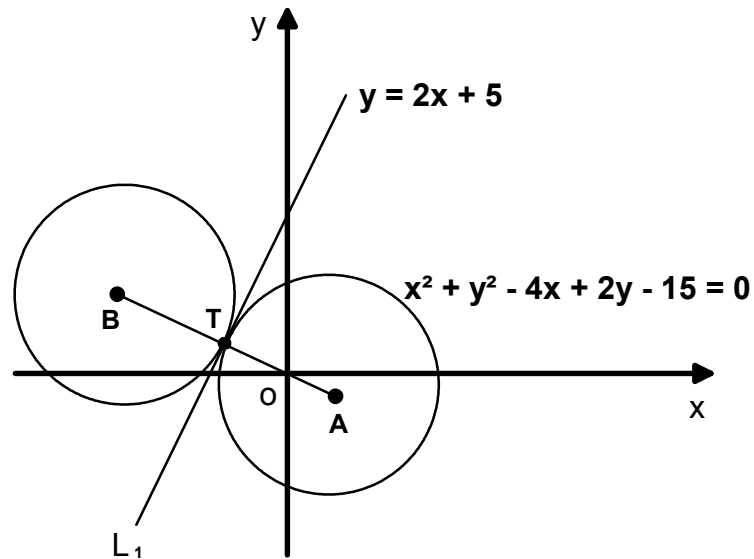


1. The diagram below shows a triangle OAB. A and B are the points (6,12) and (18,0) respectively. The median from A meets OB at M as shown.



- a) Show that the median, AM, has the equation $y = -4x + 36$ [3]
- b) Find the equation of the median from the origin to AB. [3]
- c) Show that the two medians meet at the point D(8,4). [2]
- d) Show that the distance of AD is twice the distance of DM. [3]
2. a) The terms of a sequence satisfy $U_{n+1} = kU_n + 5$. Find the value of k which produces a sequence with a limit of 4. [2]
- b) A sequence satisfies the recurrence relation $U_{n+1} = mU_n + 5$, $U_0 = 3$.
- (i) Express U_1 and U_2 in terms of m .
- (ii) Given that $U_2 = 7$, find the value of m which produces a sequence with no limit. [5]

3. Two circles, both with the same radius, touch externally at T as shown below.



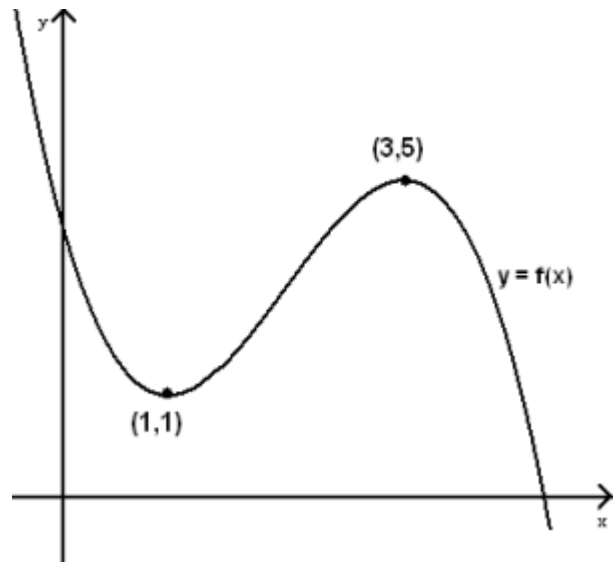
The circle with A as its centre has equation $x^2 + y^2 - 4x + 2y - 15 = 0$.
Line L_1 is the common tangent to both circles through T and has as its equation $y = 2x + 5$.

- a) Find the coordinates of T, the point of tangency. [3]
- b) Find the coordinates of B and hence write down the equation of the other circle in the diagram. [3]

4. The graph of the cubic function $y = f(x)$ is shown in the diagram.

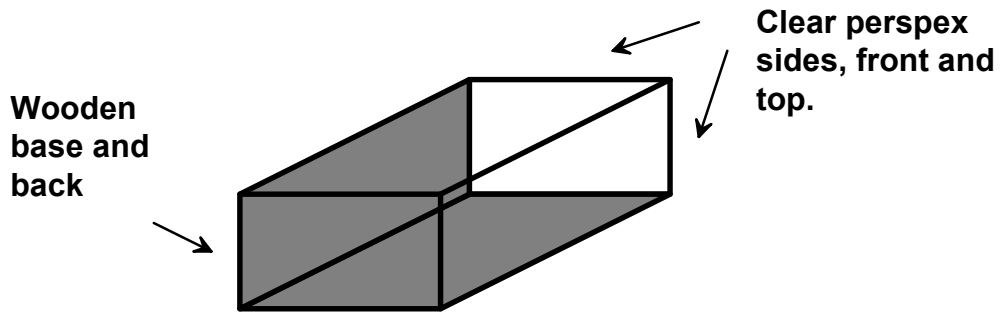
There are turning points at (1,1) and (3,5).

Sketch the graph of the derivative, $y = f'(x)$.

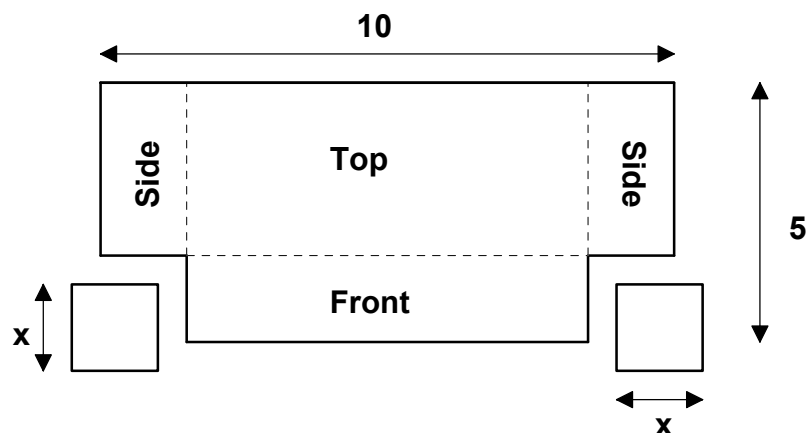


[3]

5. A museum is constructing a cabinet to hold some new exhibits. Two sides of the cabinet are made from wood (which is inexpensive and easy to obtain). The other four sides are made from a very expensive clear perspex. An illustration of the cabinet is shown below.



The four sides of perspex are to be cut from a single sheet measuring 10 feet by 5 feet. This is done by removing two squares of length x feet from the bottom corners of the sheet of perspex (see below).



- a) Show that the volume, $V(x)$, of the cabinet is given by

$$V(x) = 2x^3 - 20x^2 + 50x$$
 [3]
- b) The museum wishes to **maximise** the use of the perspex sheet. What value of x makes the cabinet of largest volume? [6]
- c) Calculate the largest volume and give your answer in cubic feet. [2]
6. The point $P(x,y)$ lies on the curve with equation $y = 6x^2 - x^3$.
- a) Find the value of x for which the gradient of the tangent at P is 12. [5]
- b) Hence find the equation of the tangent at P . [2]

7. Two functions are defined on a suitable domain as $f(x) = x^2 + a$ and $g(x) = x + 1$, where a is a constant.

a) Find the value of a given that $f(g(-2)) = -1$ [2]

b) Hence solve the equation $f(f(x)) = 2$ [5]

8. a) By expressing $\sin(3x)$ as $\sin(2x+x)$ and expanding the brackets, show that

$$\sin(3x) = 3 \sin x - 4 \sin^3 x$$
 [5]

b) Use the above relationship to solve the equation,

$$3 \sin \theta = 4 \sin^3 \theta \quad \text{where } 0 \leq \theta \leq \pi$$
 [3]

End of question paper