



Graphs of Functions: Getting Started

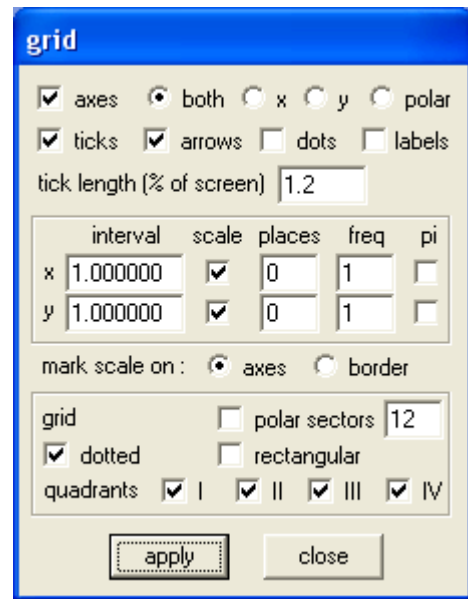
Start WinPlot as directed by your teacher.

From the menu bar, select <Window> and <2-Dim>, a new window will open. Select <Equa> and <Explicit>. Delete the default function, $x\sin(x)$ by pressing the delete key.

We will plot the graph of $f(x) = x(x - 3)^2$. You can enter a power in Winplot by using the “^” symbol which is above the 6 on the keyboard. Enter the function by typing $x(x-3)^2$. Also change the pen width to 2 and the colour to blue. Close the Inventory dialogue box which opens automatically.

From the menu bar select <View> and <Grid>.

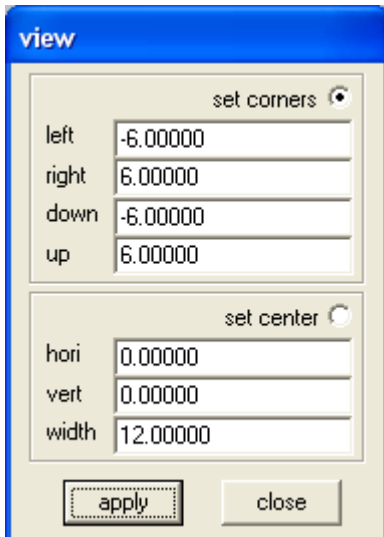
Adjust the Grid dialogue box as shown opposite.



Once you have made all the changes press **apply**. Close the Grid dialogue box.

From the menu bar select <View> then <View>.

Adjust the View dialogue box as shown below.



Apply these changes and close the View dialogue box.

Expand the window to fit the screen (centre button) and close any other dialogue boxes still open.



From the menu bar, select <View> then <Axes> and change the screen thickness to 2.

From the menu bar, select <Misc> then <Colors> and change the background to white.

You should now have the same graph which is printed on your worksheet.

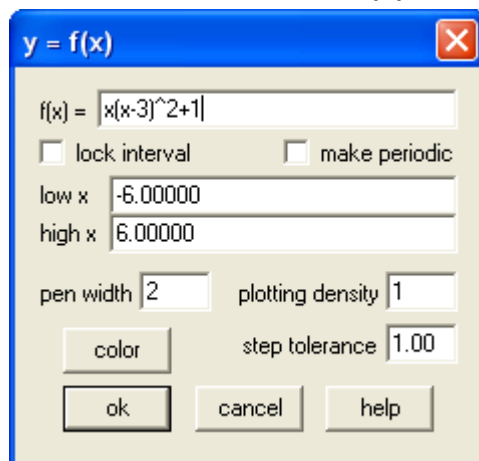
Save this graph by selecting <File> then <Save As>. Save to your MyDocuments folder using the name “startgrid”. If anything goes wrong later in the notes you can load these settings back into WinPlot by selecting <File> then <Open>.

Graphs of Functions: Graph of $f(x) + a$

To see the effect of changing the graph to $f(x) + 1$, that is, $f(x) = x(x+3)^2 + 1$ select <Equa> then <Inventory>. Click on the **dupl** button to duplicate the original function (do not replace the original as warned by WinPlot).

In the $y=f(x)$ dialogue box, change the function to $x(x+3)^2+1$ as shown below. Change the colour to purple and click **OK**.

Sketch the new function on your worksheet and label it **$f(x)+1$** .



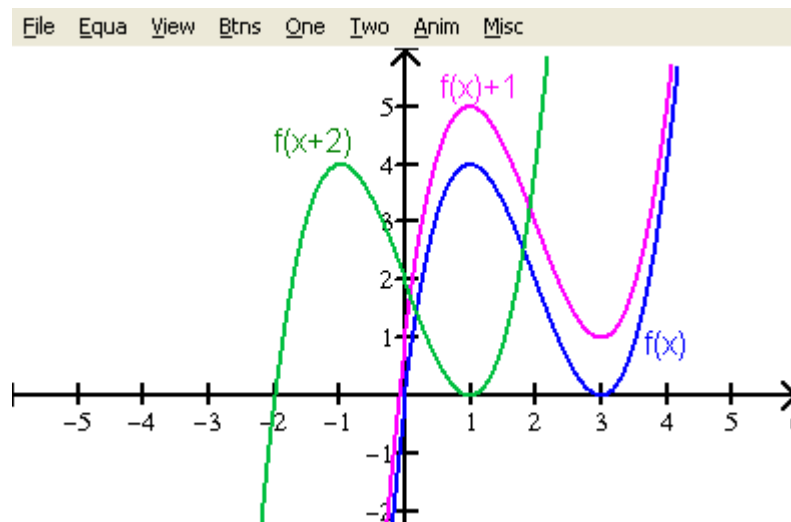
Graphs of Functions: Graph of $f(x+a)$

To see the effect of changing the function to $f(x+2)$, that is $f(x) = (x+2)((x+2)-3)^2$ select <Equa> then <Inventory>. Make sure the original function is highlighted then click the **dupl** button. Do not replace the original. Edit the function to show $(x+2)((x+2)-3)^2$ and change the colour to green. Close the Inventory dialogue box.

Compare the original blue function $f(x)$ with the new green function $f(x+2)$.

Sketch the new function on your worksheet and label it $f(x+2)$ as shown below.

Make some comments about how the graph has changed (moved).



Graphs of Functions: Graph of $-f(x)$

Before continuing we will delete the graphs of $f(x) + 1$ and $f(x + 2)$.

Select <Equa> then <Inventory>. Highlight $f(x) = (x+2)((x+2)-3)^2$ and click delete.

Highlight $f(x) = x(x-3)^2 + 1$ and click delete.

Click dupl and change the function to $f(x) = -x(x-3)^2$. Choose red as the new colour and click OK. The new graph represents $-f(x)$.

Using a new diagram on the worksheet, sketch the graph of $-f(x)$. Label the original as $f(x)$ and the new graph as $-f(x)$. Comment on what has happened to the original graph.

As before, delete the $-f(x)$ function using the <Inventory> dialogue box.

Graphs of Functions: Graph of $f(-x)$

You should only have the original graph showing on WinPlot.

We are now about to replace x with $-x$ in our function, that is $f(-x) = (-x)((-x) + 3)^2$ which could be written in a simpler form but we will leave it the way it is for convenience.

Select <Equa> then <Inventory> and duplicate the original function.

Change the function to $f(x) = (-x)((-x)+3)^2$ and select red again for the colour. Click **OK**.

Using a new diagram on the worksheet, sketch the graph of $f(-x)$. Label both graphs ($f(x)$ and $f(-x)$) and comment on how the original has changed this time.

Before continuing, delete the $f(-x)$ graph.

Graphs of Functions: Graph of $kf(x)$

We are now going to see the effect of multiplying the function by a constant. Let's try $k = \frac{1}{2}$.

Again make a copy of the original from the <Inventory> dialogue box by clicking **dupl**.

To enter $\frac{1}{2}f(x)$ change the function to $f(x) = 0.5x(x-3)^2$ and use red as the colour.

On a new diagram, sketch the new graph. Label the two graphs, $f(x)$ and $\frac{1}{2}f(x)$ and comment on what has happened to the original graph.

Delete the new graph to leave just the original as before.

Graphs of Functions: Graph of $f(kx)$

As before, duplicate the original function from the <Inventory> dialogue box.

Change the function to $f(x) = 2x(2x-3)^2$, that is $f(2x) = 2x(2x-3)^2$ and select red again.

Use a new diagram to sketch the function $f(2x)$. Label both graphs, $f(x)$ and $f(2x)$. Comment on what has happened to the original this time.

Graphs of Functions: Combining effects

Any number of these effects can be combined although things can become a little complicated. To make life easier, think of the effects one at a time. For example, to sketch the graph of $f(2x+1)$, think about $f(2x)$ first then consider what the "+1" does.

On a new diagram, make a sketch of $f(2x)$ then $f(2x+1)$ without the help of WinPlot.

Now check the new graph using WinPlot by selecting <Inventory> dialogue box. Delete all but the original function then click **dupl**.

Change the original function to $f(x) = (2x+1)((2x+1)-3)^2$ and choose a suitable colour.

Hopefully Winplot will now draw the same graph as your sketch!

Practical exercise

- 1
 - a) Using the same settings as before, use WinPlot to sketch $f(x) = x^4 - 5x^2 + 4$.
 - b) Draw the graph on squared paper.
 - c) Sketch $-f(x)$ on the same diagram THEN check your answer using WinPlot.
 - d) Draw the original again on squared paper then sketch $f(x-2)$ on the same diagram.
 - e) What can you say about the graphs of $f(x)$ and $f(-x)$ for this function?

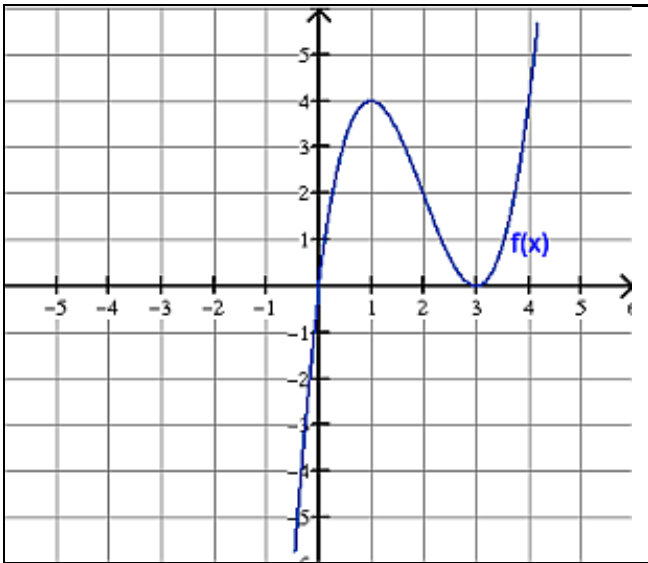
2.
 - a) Using the same settings as before, use WinPlot to sketch $f(x) = 2^x - 1$
 - b) Copy the graph onto squared paper.
 - c) On the same diagram, make a sketch of $f(-x)$ THEN check your graph using WinPlot
 - d) Use WinPlot to show you $f(x+2)$. Did the graph move 2 units back in the x-direction?

3. Trigonometric graphs are governed by the same rules. WinPlot uses Radian measure for the x-axis (instead of degrees) but you'll still see the familiar shapes even if you haven't used Radians before.
 - a) Ask WinPlot to draw the graph of $f(x) = 2 \sin(x)$ using the same settings as before.
 - b) Make 2 copies of this graph.
 - c) On one copy, draw the graph of $f(x) = 2 \sin(x) + 2$.
 - d) On the other copy, draw the graph of $f(x) = 2 \sin(-x)$.
 - e) Now try to draw the graph of $f(x) = 2 \sin(-x) + 2$. Use WinPlot to check your graph.

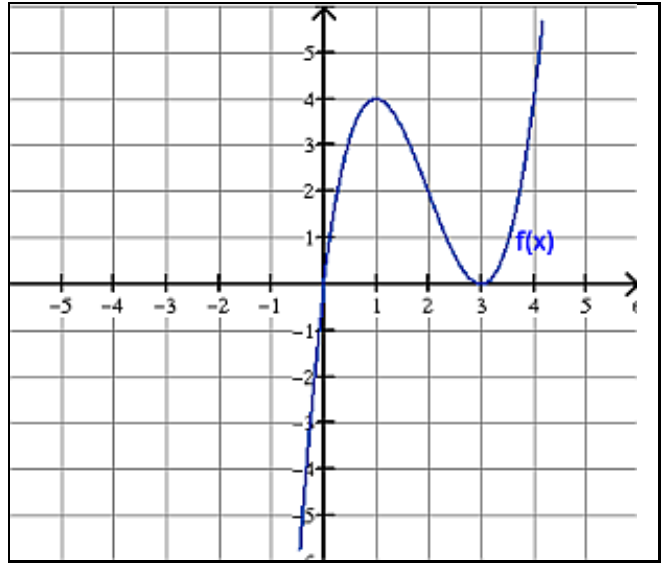
4. You can use WinPlot to practice on any graph provided you know the original function. Here are some functions you can play around with using the same grid settings as before. Try to predict each effect before you ask WinPlot to draw it. For example, try $-f(x)$, $f(-x)$, $f(x+1)$ etc.. Have fun!
 - a) $f(x) = 3 - 2x - x^2$
 - b) $f(x) = \frac{1}{x}$ (use x^{-1} or $1/x$ in WinPlot)
 - c) $f(x) = (x-1)^2(x-4)$
 - d) $f(x) = 2x^3 - 4x^2 + 2x$
 - e) $f(x) = 1 - 2^x$

WinPlot is a completely FREE graphical program which can be downloaded from the internet and used at home. The maths page on the school's web site has a link to WinPlot and other related material.

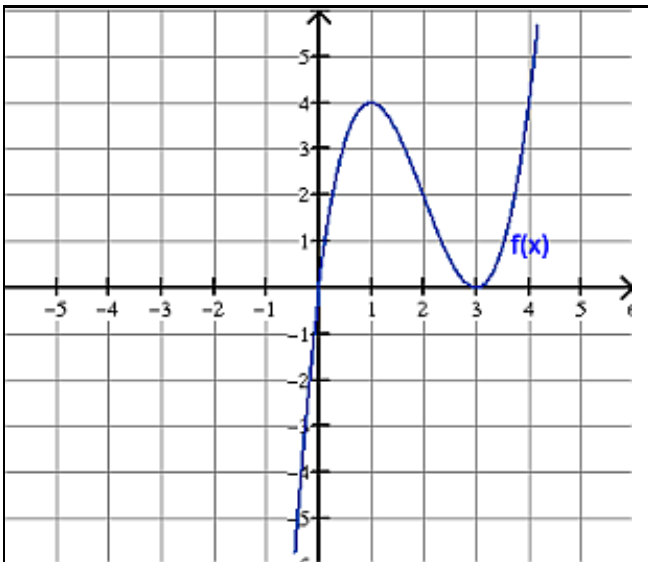
Graphs of Functions: Worksheet for sketches and comments.



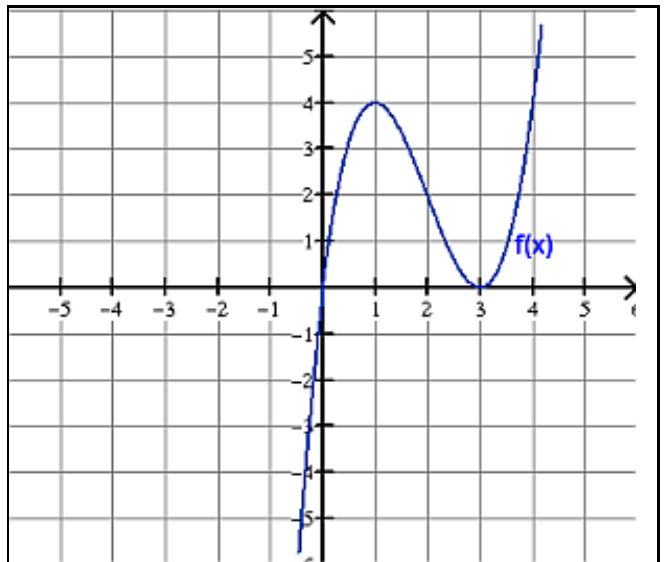
$f(x)$, $f(x+1)$ and $f(x+2)$



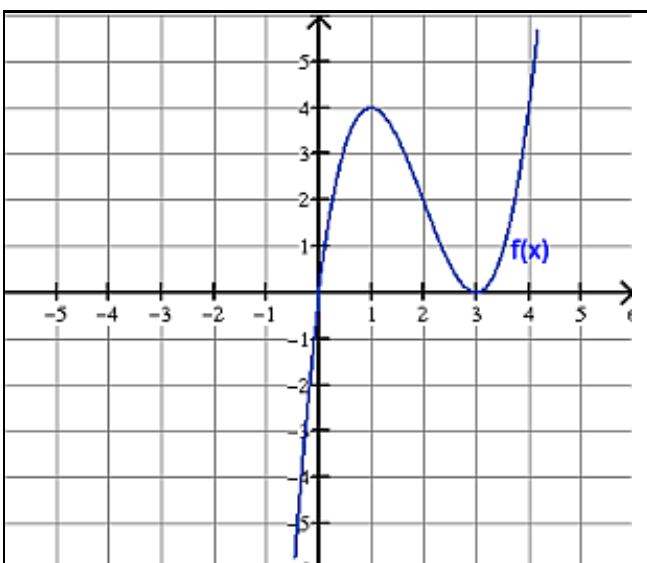
$f(x)$ and $-f(x)$



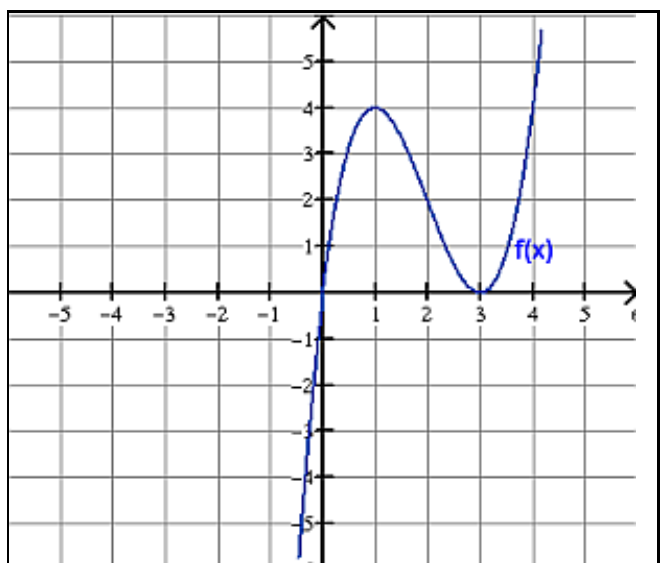
$f(x)$ and $f(-x)$



$f(x)$ and $\frac{1}{2}f(x)$



$f(x)$ and $f(2x)$



$f(x)$, $f(2x)$ and $f(2x+1)$