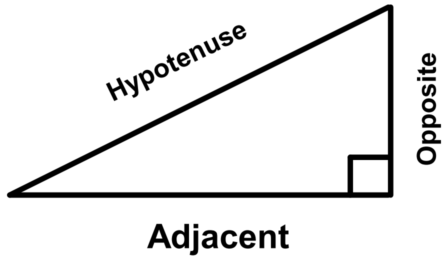
Trigonometry

1) "SOH, CAH, TOA!"

$$\sin A = \frac{OPPOSITE}{HYPOTENUSE}$$

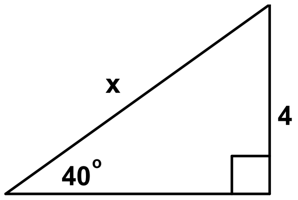
$$\cos A = \frac{ADJACENT}{HYPOTENUSE}$$

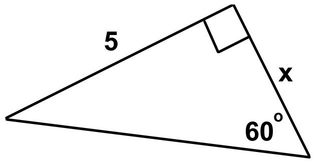
$$\tan A = \frac{OPPOSITE}{ADJACENT}$$

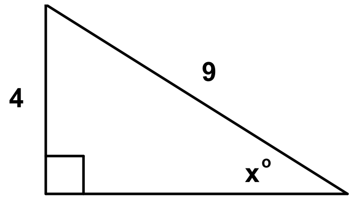


2) Using the 3 Trigonometric Functions

Examples: Find the value of x, in each of the following:-







$$\sin A = \frac{OPP}{HYP}$$

$$\sin 40^o = \frac{4}{x}$$

$$x = \frac{4}{\sin 40^o}$$

= 6.22

$$\tan A = \frac{OPP}{ADJ}$$

$$\tan 60^o = \frac{5}{x}$$

$$x = \frac{5}{\tan 60^o}$$

= 2.89

$$\sin A = \frac{OPP}{HYP}$$

$$\sin A = \frac{4}{9}$$

$$\sin A = 0.444$$

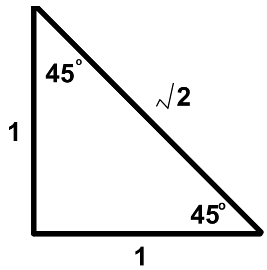
A = 26.4^0

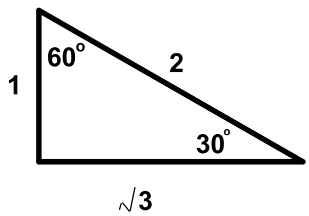
3) Exact Values

(C)

From the special triangles ....

We can write down the following **exact values**...





$$\sin 45^o = \frac{1}{\sqrt{2}} \qquad \sin 60^o = \frac{\sqrt{3}}{2} \qquad \sin 30^o = \frac{1}{2}$$

$$\cos 45^o = \frac{1}{\sqrt{2}} \qquad \cos 60^o = \frac{1}{2} \qquad \cos 30^o = \frac{\sqrt{3}}{2}$$

$$\tan 45^o = 1 \qquad \tan 60^o = \sqrt{3} \qquad \tan 30^o = \frac{1}{\sqrt{3}}$$

4) Trigonometric Identities (C)

For all values of x° .....

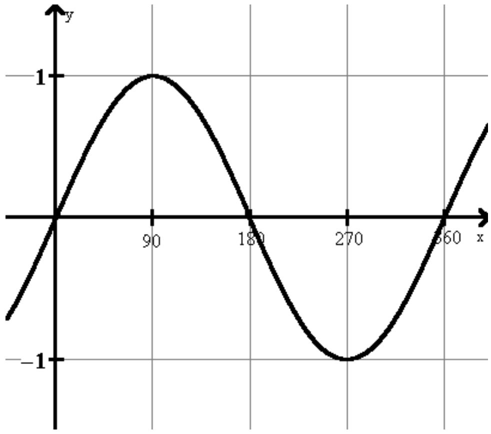
$$\boxed{\begin{array}{l}\sin^2 x + \cos^2 x = 1 \\ \\ \tan x = \dfrac{\sin x}{\cos x}\end{array}}$$

from which you have .....

$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

$$\sin x = \cos x \tan x$$

5) The Graph of Sin(x), Cos(x) and Tan(x) (C)

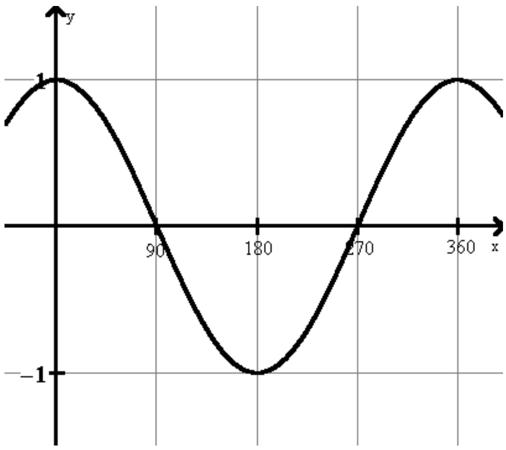


y = SIn(x)

Period = 360^0

sin(0) = 0,   sin(90) = 1

sin(180) = 0   sin (270) = -1

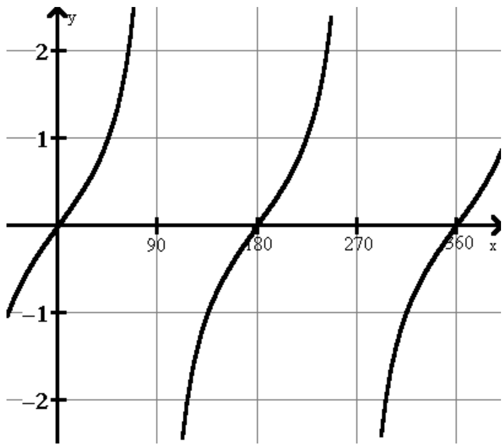


y = Cos(x)

Period = 360^0

cos(0) = 1   cos(90) = 0

cos(180) = -1   cos(270) = 0



y = Tan(x)

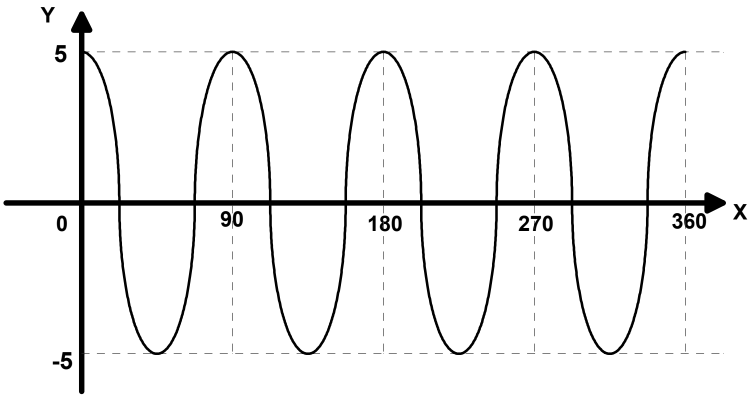
Period = 180^0

tan(0) = 0   tan(90) = ∞

tan(180) = 0   tan(270) = ∞

Notice: The **maximum** value of the **sine** and **cosine** graph is **1.**
The **minimum** value of the **sine** and **cosine** graph is **-1**.
The **tangent** graph has **no maximum** and **no minimum**.

6) <u>More Trigonometric Graphs</u> (C)

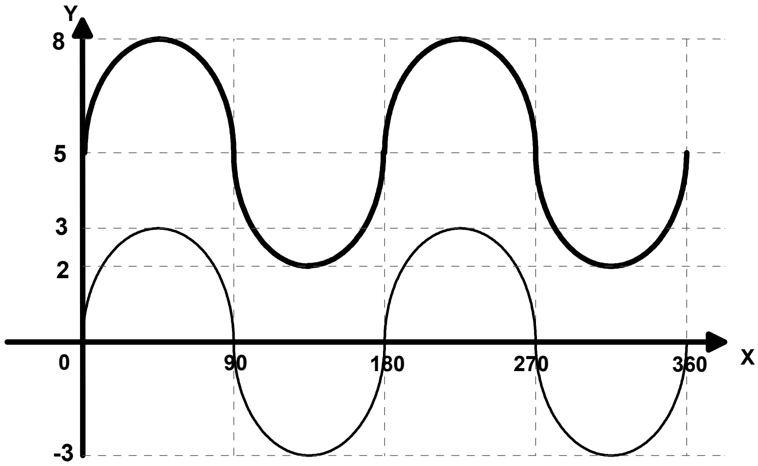


Example:-

y = 5cos(4x)

Period = 360/4 = 90^0

Max. value = 5
Min. value = -5



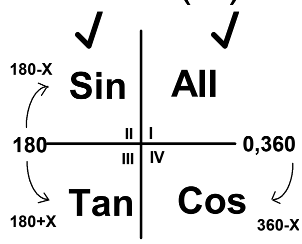
Example:-

y = 3sin(2x) + 5 (upper graph)
y = 3sin(2x) (lower graph)

For y = 3sin(2x) + 5,
Max. value = 3(1) + 5 = 8
Min value = 3(-1) + 5 = 2

Period = 360/2 = 180^0

7) <u>Using The Four Quadrants</u> (C)



Example: Solve for x, **sinx = 0.5**, for $0 < x < 360^0$.

From the diagram opposite, the solutions are in the 1st and 2nd quadrants (where sinx is positive).
The calculator will always give the answer in the first quadrant only. Use the SHIFT key and the $\boxed{\text{sin}}$ key to obtain:-

x = <u>30^0</u> and x = 180 - 30 = <u>150^0</u>

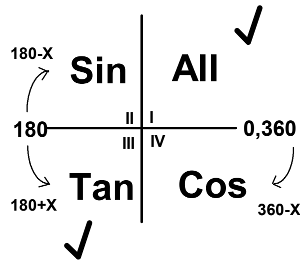
Notice that you may only use 180 or 360 when calculating solutions in other quadrants.

Remember that the 4 quadrant diagram shows where sinx, cosx and tanx are always positive (compare with graphs on previous page).

Look closely at the next two examples on the following page.

8) **More examples of Quadrant work** (C)

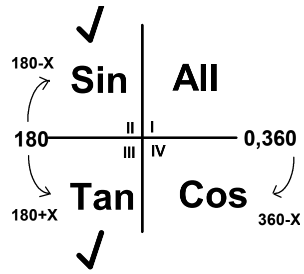
Example: Solve for x, tanx = 0.453, for $0 \le x \le 360^o$.



Answer: From the diagram, the two solutions are in the 1st and 3rd quadrants.

x = <u>24.4</u>^{0} and x = 180 + 24.4
= <u>204.4</u>^{0}

Example: Solve for x, cosx = -0.321, for $0 \le x \le 360^o$.

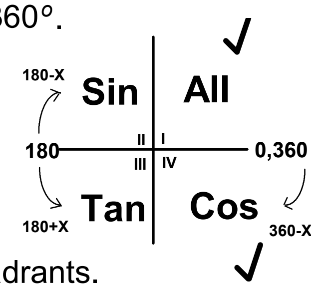


Answer: From the diagram, the two solutions are in the 2nd and 3rd quadrants.

x = 180 - 71.3 and x = 180 + 71.3
= <u>108.7</u>^{0} = <u>251.3</u>^{0}

9) **Trigonometric Equations** (C)

Example: Solve the equation $2\cos x - \sqrt{3} = 0$ for $0 \le x \le 360^o$.



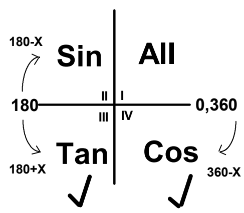
Answer:

$$2\cos x - \sqrt{3} = 0$$
$$2\cos x = \sqrt{3}$$
$$\cos x = \frac{\sqrt{3}}{2} = 0.866$$

From the diagram, the solutions are in the 1st & 4th quadrants.

x = <u>30</u>^{0} and x = 360 - 30 = <u>330</u>^{0}

Example: Solve the equation $1 + 2\sin x = 0$, for $0 \le x \le 720^o$.



Answer:

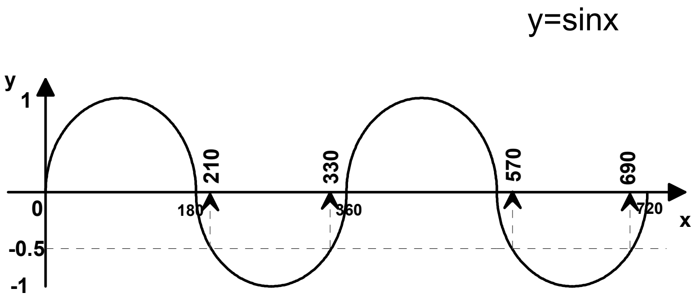
$$1 + 2\sin x = 0$$
$$2\sin x = -1$$
$$\sin x = -\tfrac{1}{2} = -0.5$$

From the diagram, the solutions are in the 3rd & 4th quadrants.

x = 180 + 30 = <u>210</u>^{0} and x = 360 - 30 = <u>330</u>^{0}

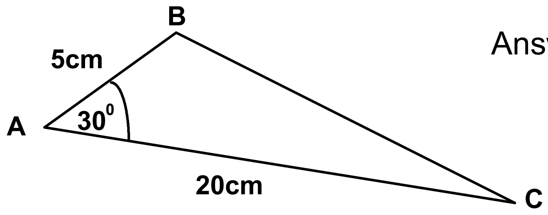
For the other two solutions,-

x = 210 + 360 & 330 + 360
= <u>570</u>^{0} <u>690</u>^{0}



10) **Area of a Triangle** (C)

Example: Find the Area of the triangle drawn below.



Answer: Area(triangle) = $\frac{1}{2}bc\sin A$

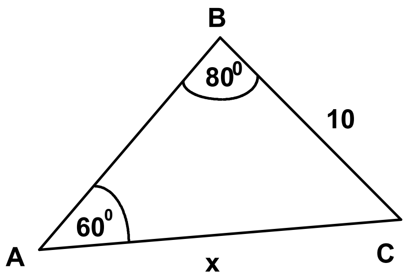
= 0.5 x 20 x 5 x sin30

= <u>25 cm^2</u>

11) **The Sine Rule** (C)

Example: Find the value of x in the triangle opposite.

$$\underset{\checkmark}{\overset{\checkmark}{\frac{a}{\sin A}}} = \underset{\checkmark}{\overset{?}{\frac{b}{\sin B}}} = \frac{c}{\sin C}$$ (Sine Rule)



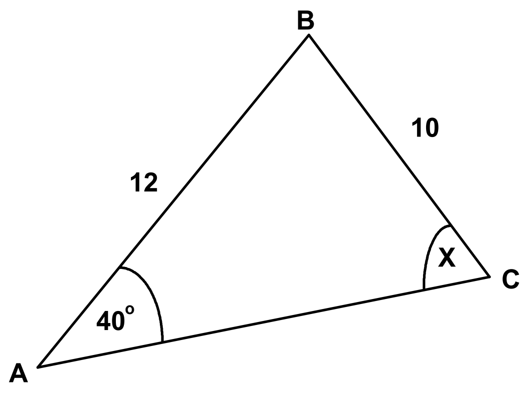
$$\frac{10}{\sin 60} = \frac{x}{\sin 80}$$

xsin60 = 10sin80

$$x = \frac{10\sin 80}{\sin 60} = \underline{11.4}$$

Example: Find the angle x in the triangle opposite,-

$$\underset{\checkmark}{\overset{\checkmark}{\frac{a}{\sin A}}} = \frac{b}{\sin B} = \underset{?}{\overset{\checkmark}{\frac{c}{\sin C}}}$$



$$\frac{10}{\sin 40} = \frac{12}{\sin x}$$

$$10\sin x = 12\sin 40$$

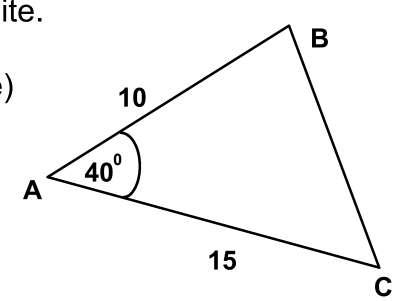
$$\sin x = \frac{12\sin 40}{10}$$

sinx = 0.771

x = <u>50.5^0</u>

12) **The Cosine Rule** (C)

Example: Find the size of BC in the triangle opposite.



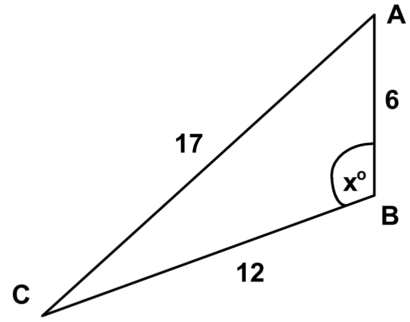
Answer: $a^2 = b^2 + c^2 - 2bc\cos A$ (Cosine Rule)

$BC^2 = 15^2 + 10^2 - 2(15)(10)\cos 40$

$BC^2 = 225 - 229.8$

$BC = \sqrt{95.2} = 9.76$

Example: Find the angle x in this triangle.



Answer: $b^2 = c^2 + a^2 - 2ca\cos B$

$17^2 = 6^2 + 12^2 - 2(6)(12)\cos x$

$289 = 180 - 144\cos x$

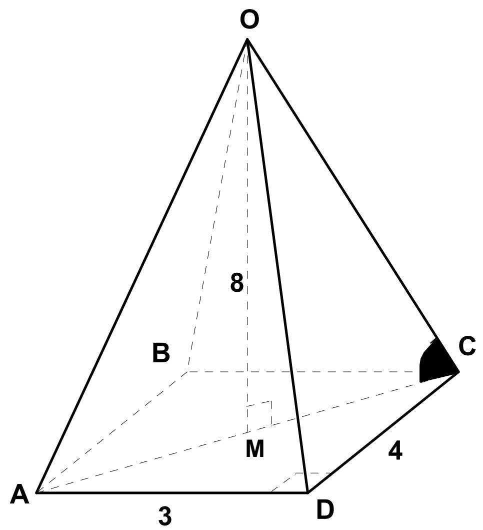
$\cos x = \dfrac{180 - 289}{144} = -0.826$

$x = 145.7^o$ (since angle is in the 2nd quadrant)

13) **3-D Drawings** (C)

This involves no new work however, a good imagination is required!

Example: In the solid below, find the length of AC and the angle ACO.



Using Pythagoras,

$AC^2 = AD^2 + DC^2$
$AC^2 = 3^2 + 4^2$
$AC = \sqrt{25} = 5$

In ΔOMC, M is a right angle & $MC = \frac{1}{2}AC$.

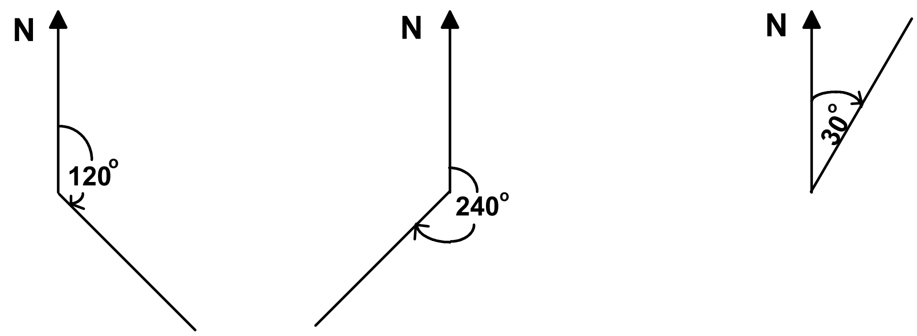
$\tan C = \frac{MO}{CM} = \frac{8}{2.5} = 3.2$

So C = 72.6^0

14) **3 Figure Bearings** (C)

Note: All three figure bearings are measured from the **North** in a **clockwise** direction (a negative rotational direction).

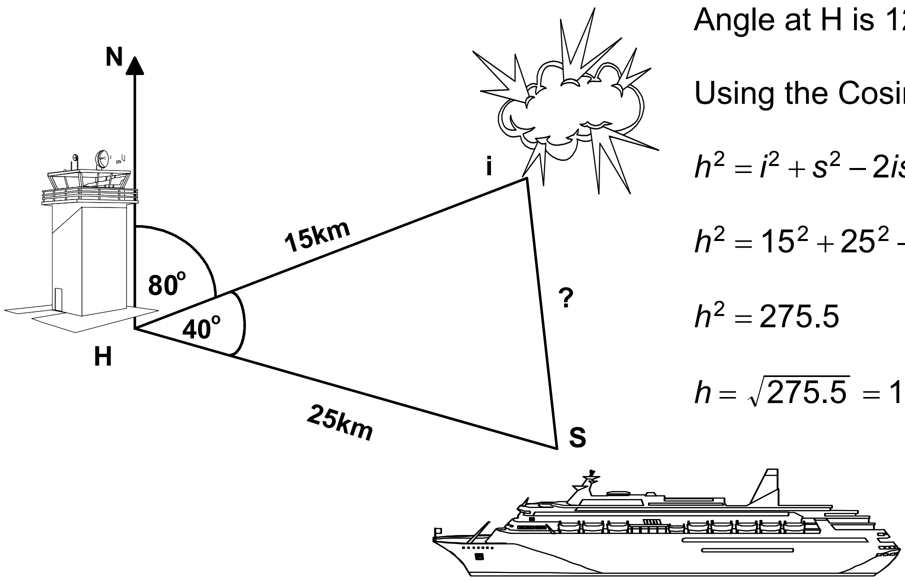
Examples: Sketch bearings of a) 120^0 b) 240^0 c) 030^0



15) **Working with Bearings** (C)

Example: A ship (S) lies on a bearing of 120^0 from a Control Tower (H) which is 25km away. A storm (i) is known to be on a bearing of 080^0 from the Control Tower. If the distance between the storm and the Control Tower is15km how far is the ship from the storm?

Answer: A good sketch is always required!



Angle at H is 120 - 80 = 40^0

Using the Cosine Rule,

$$h^2 = i^2 + s^2 - 2is\cos H$$

$$h^2 = 15^2 + 25^2 - 2(15)(25)\cos 40$$

$$h^2 = 275.5$$

$$h = \sqrt{275.5} = 16.6km$$

So the ship is only <u>16.6km</u> from the storm.